Informational Substitutes

Definitions and Design

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Motivation

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. . . but tricky!

- bicylce sale data / helmet sale data, to safety researcher

Broad research question

Can we:

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- discover evidence that these are **natural**?
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Challenges:

- Information is more complex and structured than items.
- Applications may not have been apparent.

This paper

- Defines **informational S&C**.
  (will compare to prior attempt in Börgers et al 2013)
- Application to prediction markets.
  Characterize **market equilibria** in terms of S&C.
- Sundry observations and results (including algorithmic).
  Will present **Value of Information** plots,
  design considerations.

Part 1: Developing the definitions

Outline for part 1

Goal: define substitutability as "diminishing marginal value of information".

1. What is the value of information?
2. What is a "marginal unit" of information?
3. The definitions.

Then: will compare approach of Börgers, Hernando-Veciana, Krähmer 2013.

The value of information

We focus on a **single-agent decision problem**:

1. Nature draws signals $A_1, \ldots, A_m$ and an event E jointly from a known prior p.
2. Agent observes A: subset or "garbling" of the signals.
3. Agent selects a decision d.
4. Agent receives utility $u(d, e)$ where $E = e$.

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Value of information:

$$\mathcal{V}^{u,p}(A) = \mathop{\mathbb{E}}_{a \sim A} \max_{d} \mathbb{E}\left[u(d, E) \mid A = a\right].$$

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2. Release some **deterministic** function of the signals (amounts to **pooling** some signal outcomes).
3. Release some randomized function, *i.e.* **garbling** of the signals.

Marginal units of information (cont)

These lead to three **signal spaces**:

1. All subsets of $\{A_1, \ldots, A_n\}$.
2. All deterministic functions (poolings) of $\{A_1, \ldots, A_n\}$.
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Each space is a **lattice** partially ordered by informativeness.

(2) Very close to Aumann's partition model; (3) to Blackwell ordering.

The definitions (weak)

Signals $A_1, \ldots, A_n$ are **weak substitutes for** u if:
for any subsets S, S', T with $S \subseteq S'$,

$$\mathcal{V}^{u,p}(S \cup T) - \mathcal{V}^{u,p}(S) \geq \mathcal{V}^{u,p}(S' \cup T) - \mathcal{V}^{u,p}(S').$$

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Signals $A_1, \ldots, A_n$ are **weak complements for** u if:
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"marginal value of T increases given more information"

This is just sub- and super-modularity of $\mathcal{V}^{u,p}$.

The moderate and strong definitions

What did we do in the weak case?

- Extended $\{A_1, \ldots, A_n\}$ to a space of signals (*i.e.* $S \subseteq \{1, \ldots, n\}$) **partially ordered by informativeness**.
- Defined substitutes as:
  If S is less informative than S', then the same signal adds more marginal value to S than to S'.

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To get moderate / strong, do the same with more "fine" signal spaces: the "deterministic" / "garblings" settings.

Discussion and prior work

Börgers et. al (2013)'s definition: what we call "universal weak substitutes":

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Drawback 1: **Universality is far too restrictive.**

- All universal weak subs have "almost trivial" structure.
- All universal moderate or strong subs are trivial.
- Meanwhile, can extend specialized defs to classes of S&C.

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Drawback 2: **Weak signals are often too permissive.**
e.g. can sometimes make them behave as complements by withholding some information.

Part 2: Prediction Market results (very briefly)

Prediction markets

In a prediction market:

- Nature draws event E and signals $A_1, \ldots, A_m$ from a known prior
- Agents observe private signals buy and sell shares in securities tied to an event E
- After the market, $E = e$ is observed and shares pay out
- We consider: centralized market maker (*e.g.* Hanson 2003, ..., Ostrovsky 2012)

A market instance is a set of agents, signals each observes from $\mathcal{L}(A_1, \ldots, A_n)$, and order of trading.

Prior results on prediction markets

Ostrovsky (2012):

> *This paper shows that, for a broad class of securities, information in dynamic markets with partially informed strategic traders* **always gets aggregated**.

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Only known for very special cases (Chen et al 2010, Gao et al 2013).

Prediction market result

- If signals are strong substitutes: For every market instance, in **every** Bayes-Nash equilibrium, traders **truthfully reveal information "as early as possible"**.
- Otherwise: there exists a market instance where **no BNE** has this property.

Prediction market result

- If signals are strong substitutes: For every market instance, in **every** Bayes-Nash equilibrium, traders **truthfully reveal information "as early as possible"**.
- Otherwise: there exists a market instance where **no BNE** has this property.

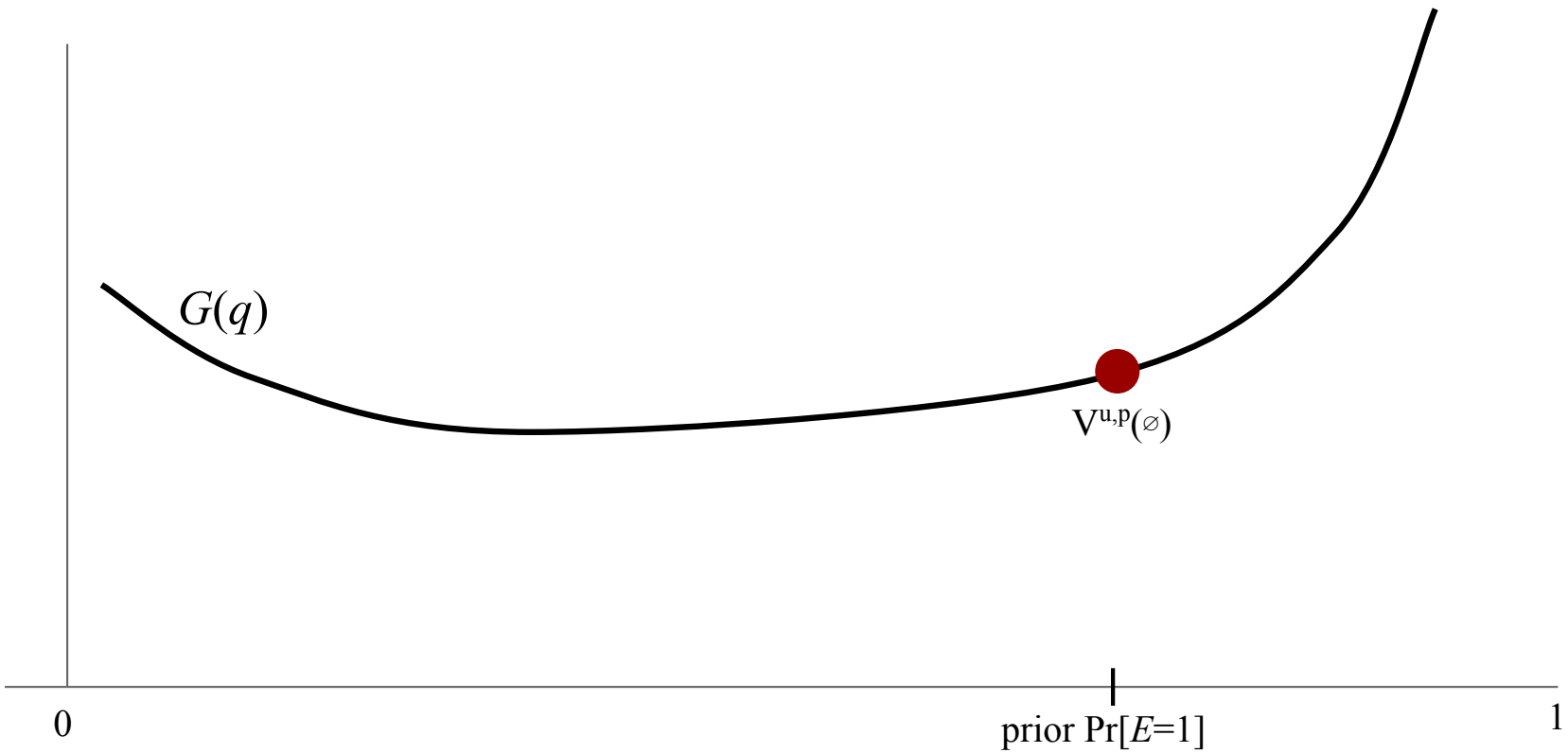
- If signals are strong complements: for every market instance, in **every** perfect Bayesian equilibrium, traders **delay "as long as possible" before revealing**.
- Otherwise: there exists a market instance where **no PBE** has this property.

Part 3: Value-of-Information plots, intuition, design

VOI Plots

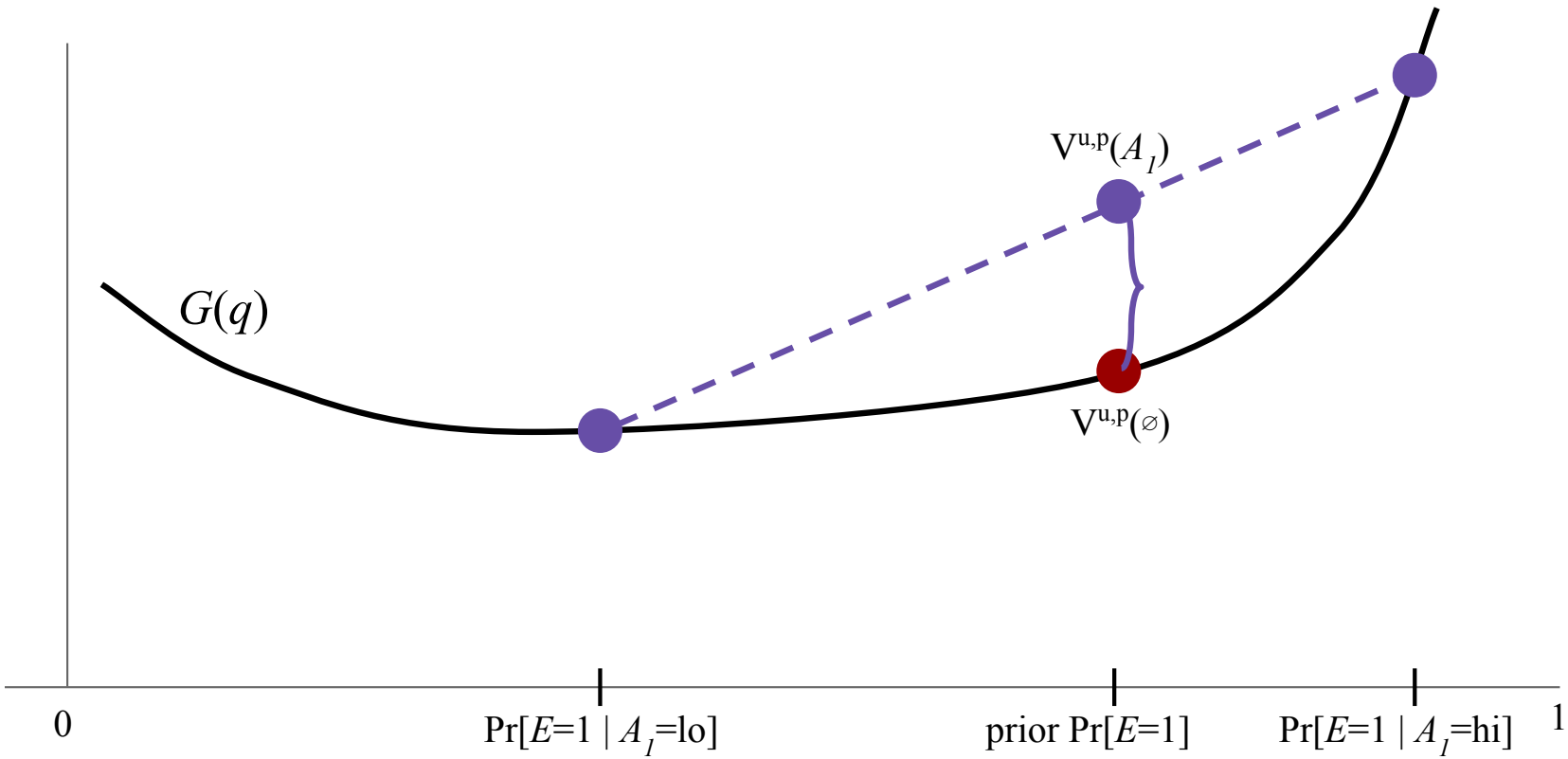
Given $u(d, e)$ where nature's event E is **binary**:

- Plot probability q of $E = 1$ on the x-axis
- Plot $G(q) = \max_d \mathbb{E}_{e \sim q}\, u(d, e)$.
- In particular, $G(\text{prior}) = \mathcal{V}^{u,p}(\emptyset)$.



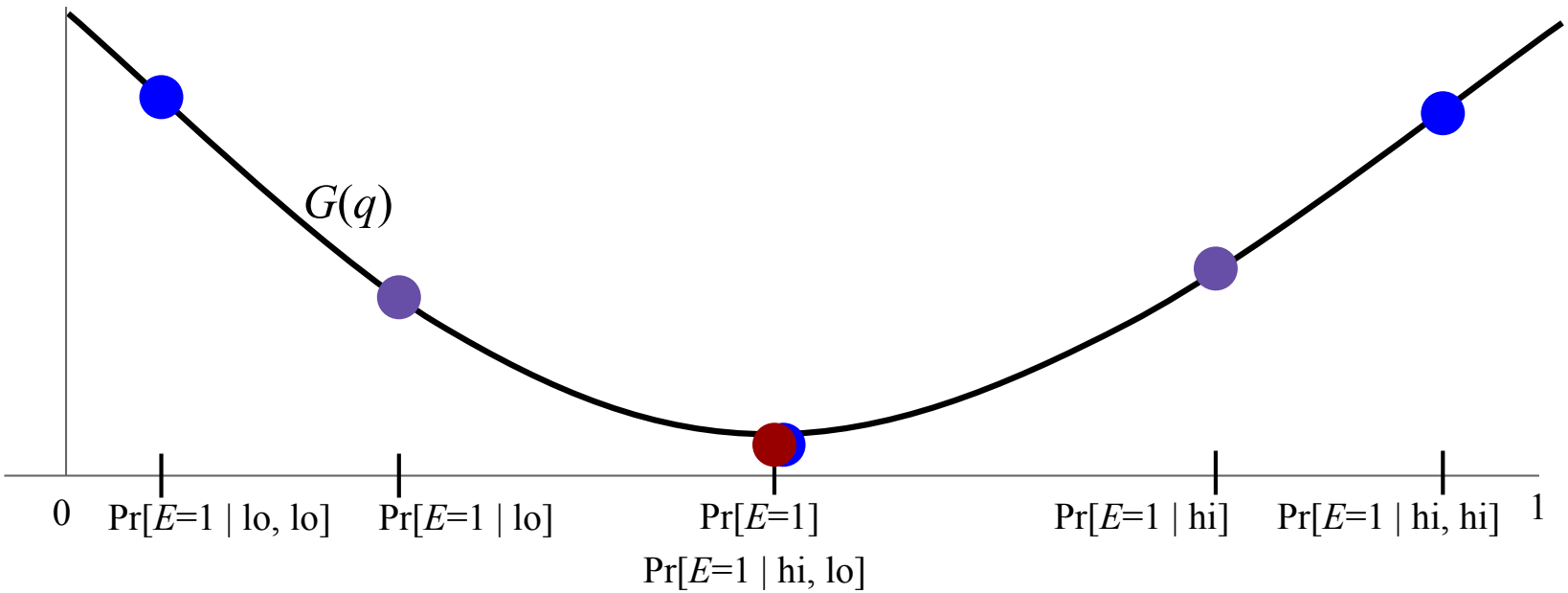
VOI Plots continued

- Now locate G(posteriors), average to get $\mathcal{V}^{u,p}(A_1)$.
- Purple brace = marginal value of A_1 over prior.



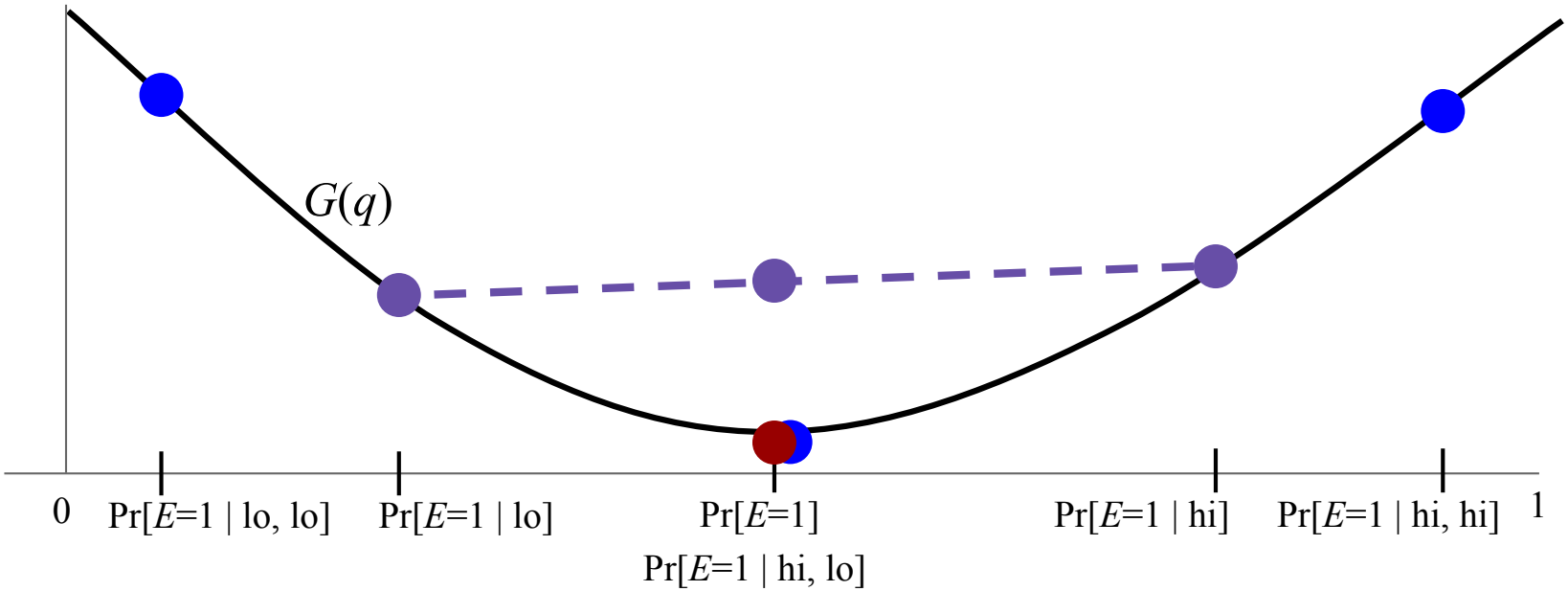
Designing for substitutes

- lots of curvature near prior = large initial value
- little curvature farther out = small later marginal value



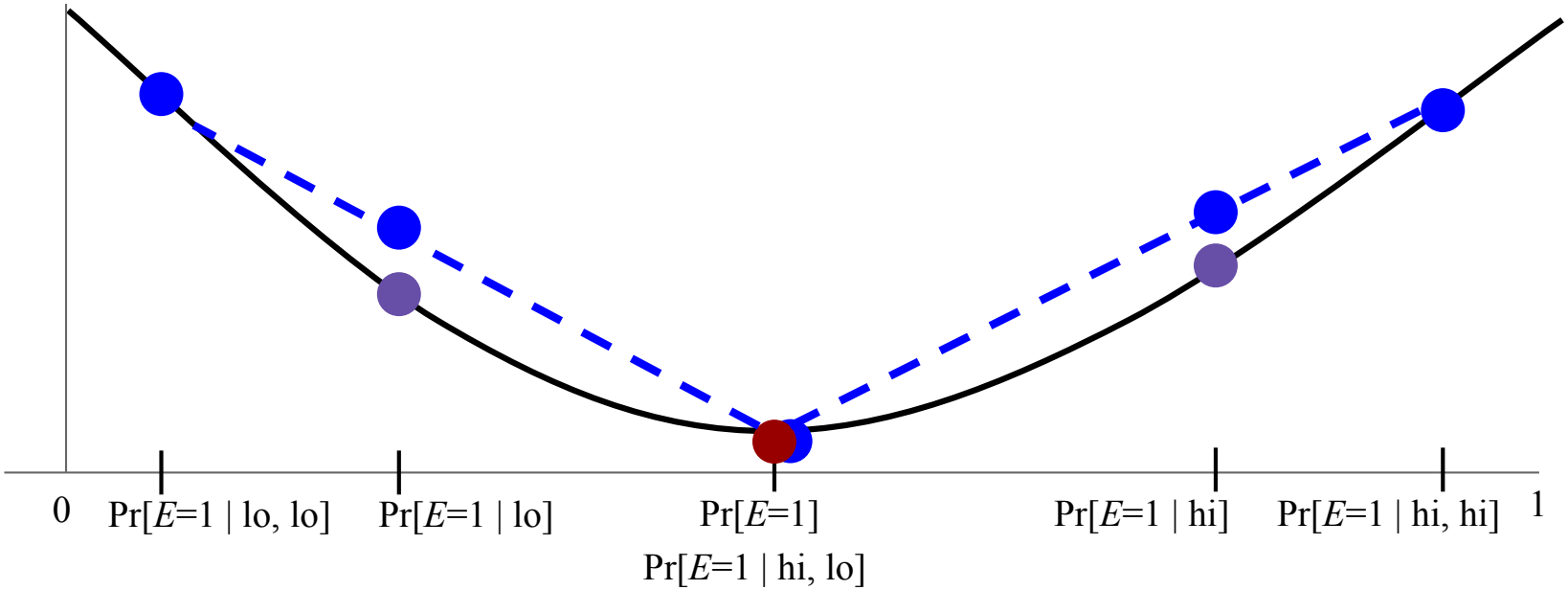
Designing for substitutes

Marginal value of first signal over the prior:



Designing for substitutes

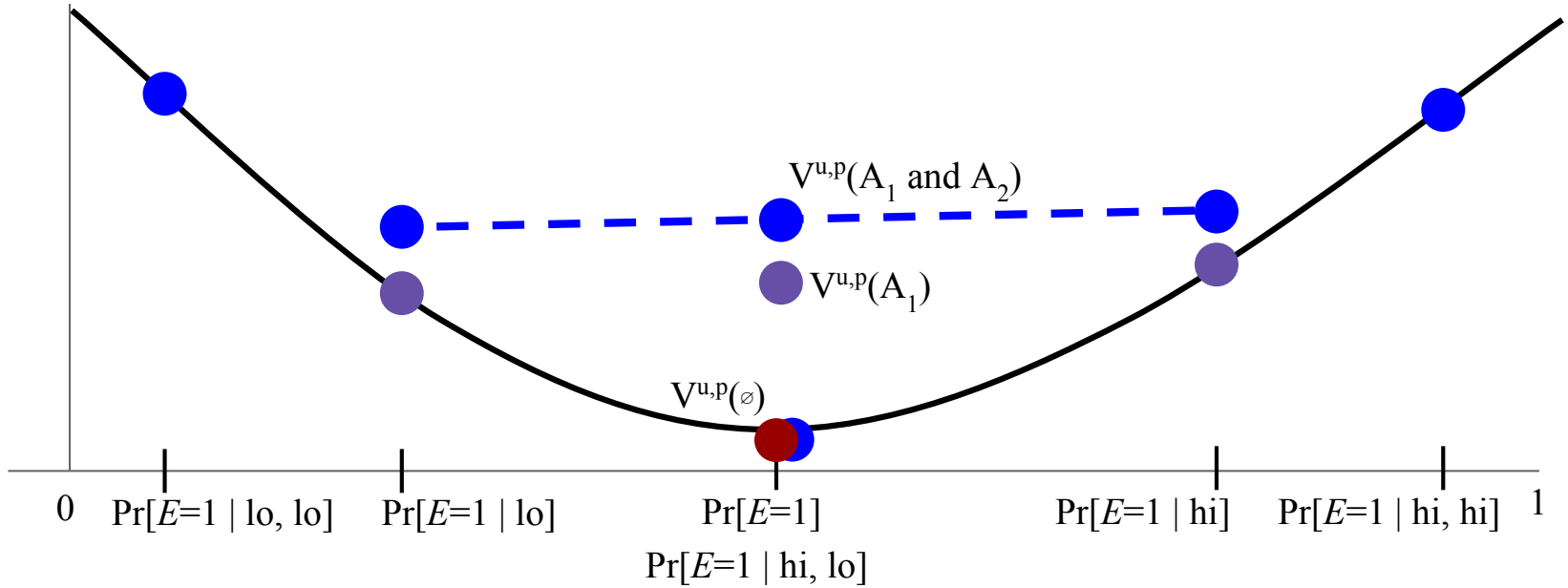
For each outcome of the first signal, marginal value of the second:



Designing for substitutes

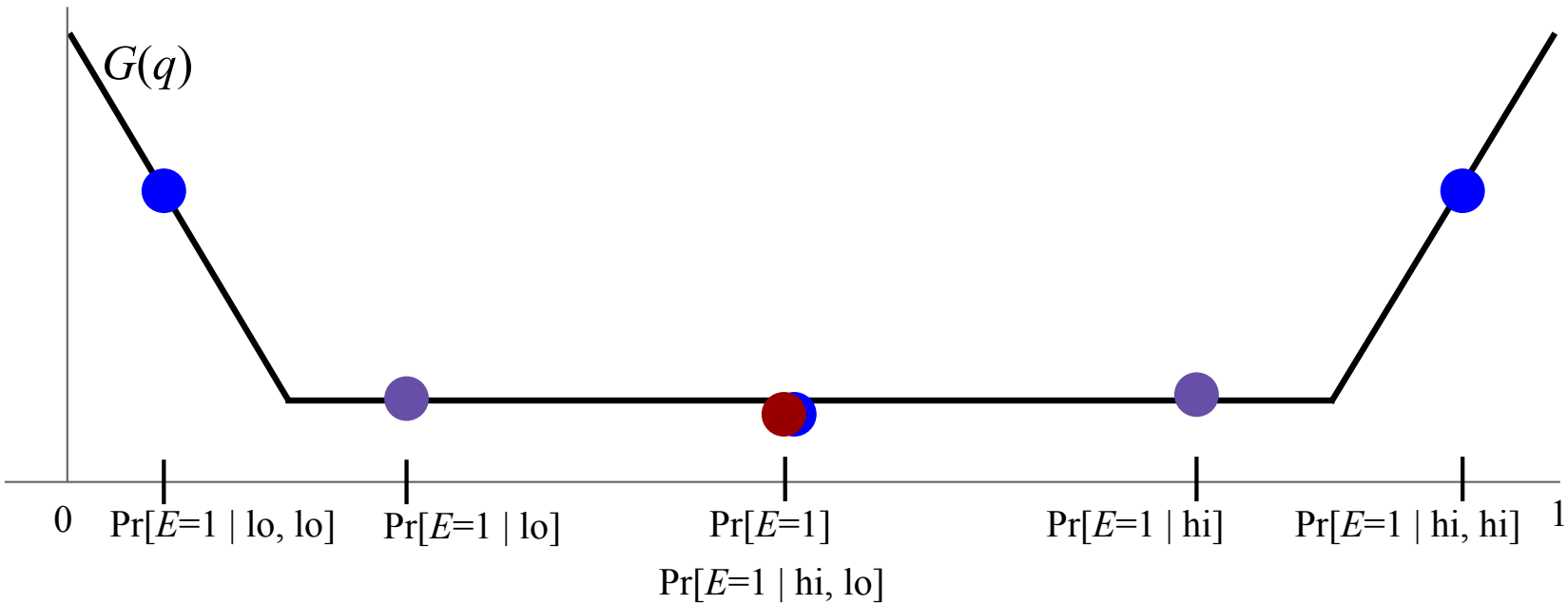
Average marginal value of 2nd signal is smaller than 1st

$\Longrightarrow$ substitutes!



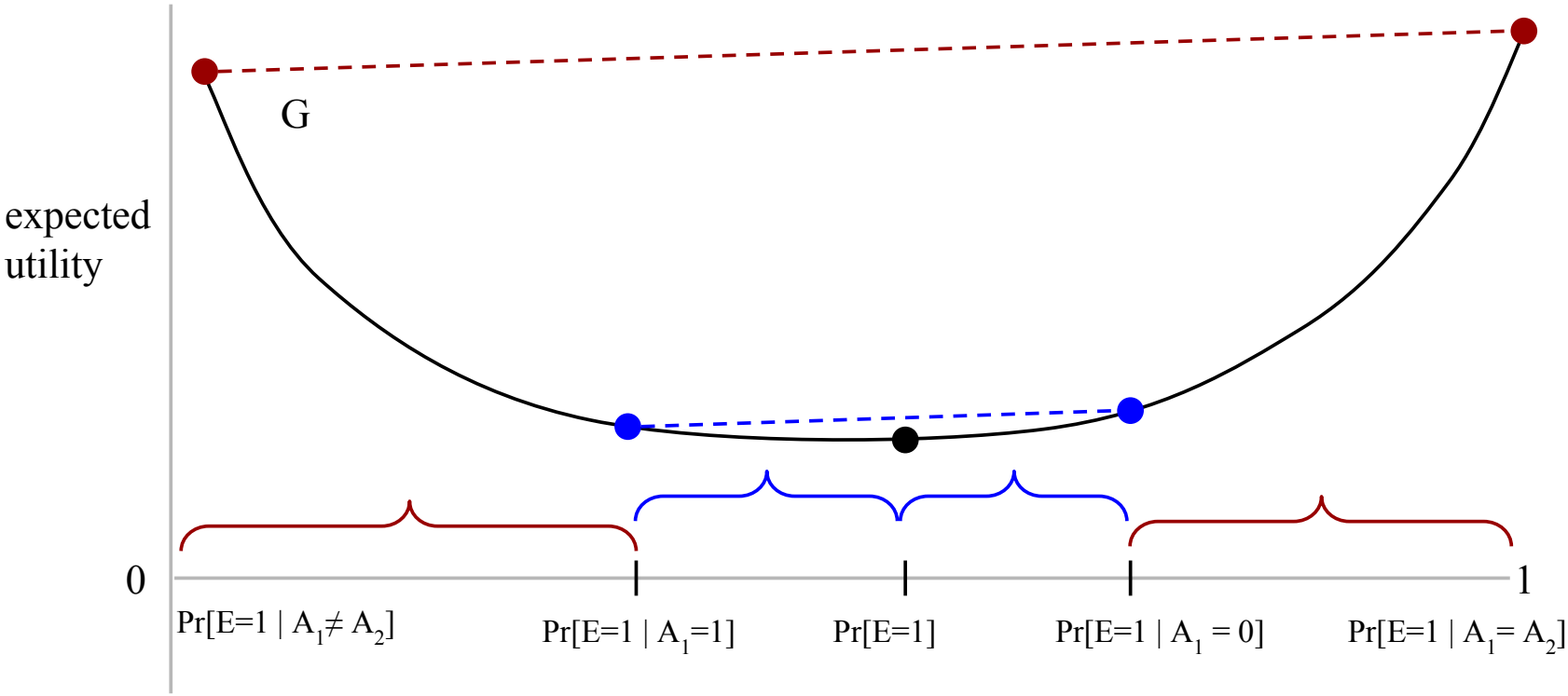
Substitutes are fragile

Even for information structures that seem substitutable, the **wrong utility function** (scoring rule) can destroy substitutability:



Complements are robust

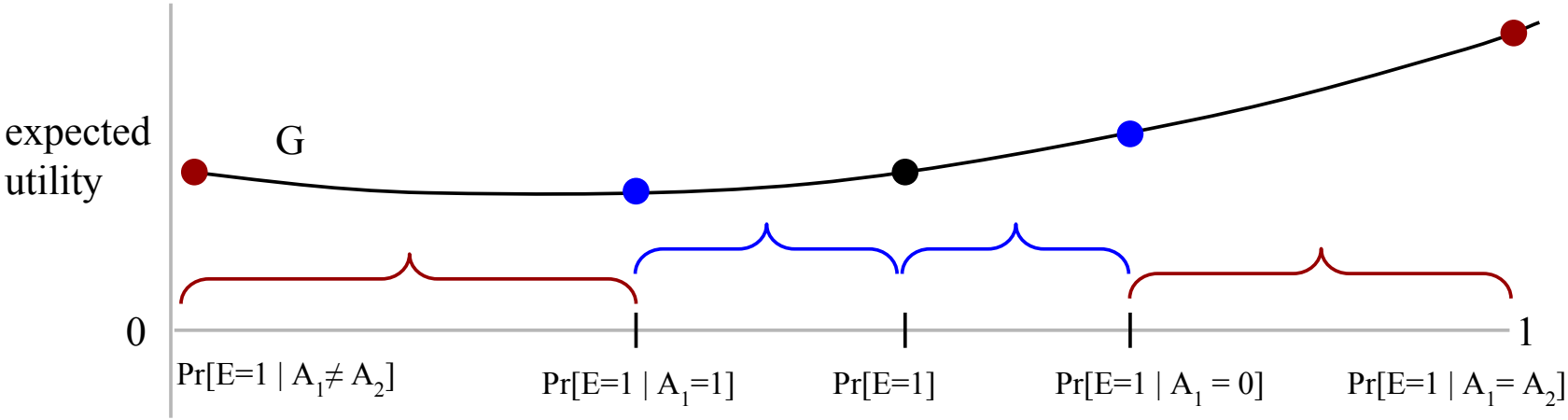
How should one design to reduce complementarities here?



"Decrease curvature away from prior"

Complements are robust

But we can only "flatten" so far!
(Also: this reduces incentives in general, must scale up to compensate)



Thank you!

Questions / Discussion?