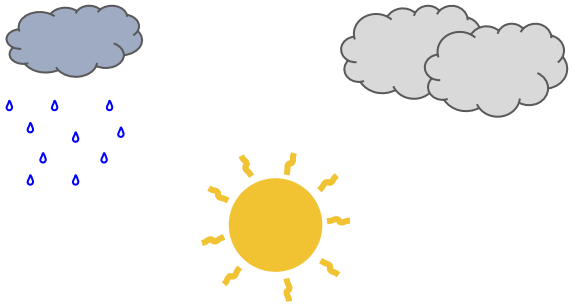
Prediction Market Equilibria via Substitutes and Complements



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INFORMS
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Outline

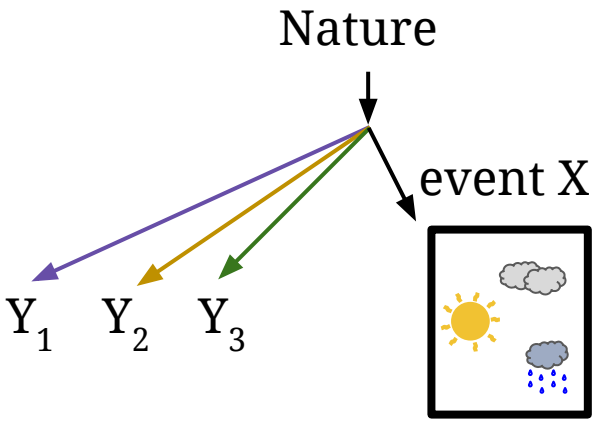
1. Background on prediction markets
2. Define substitutes and complements
3. Equilibria
4. Design of prediction markets

Information (in this talk)

Random variables X, Y_1, …, Y_n jointly distributed, known prior. (finite set of outcomes)

We care about X.

Y_i = "signal" (reveals info. about X).



Proper scoring rules

1. Known prior p on X

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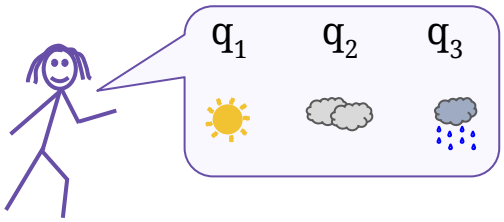
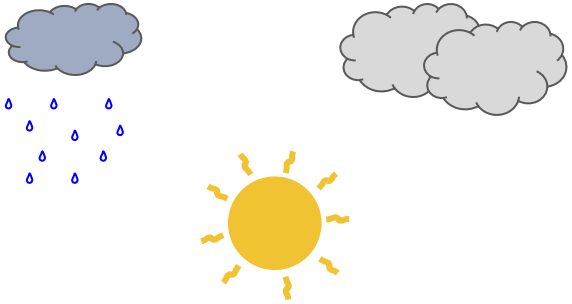
2. Agent selects **prediction q**



2. Nature draws x ~ p



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3. Agent gets score **S(q, x)**.

Proper scoring rule: optimal prediction is true belief.

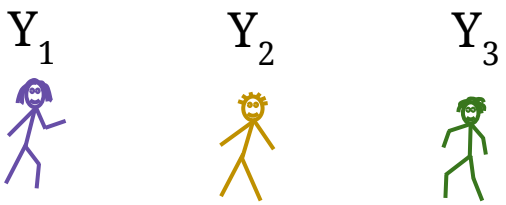
Examples: (a) $S(q,x) = \log q(x)$.

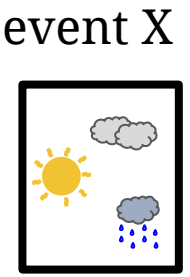
(b) $S(q,x) = 2q(x) - \sum_{x'} q(x')^2$.

Prediction markets

Each agent has a signal Y_i.

Goal: aggregate into prediction about X **quickly**.





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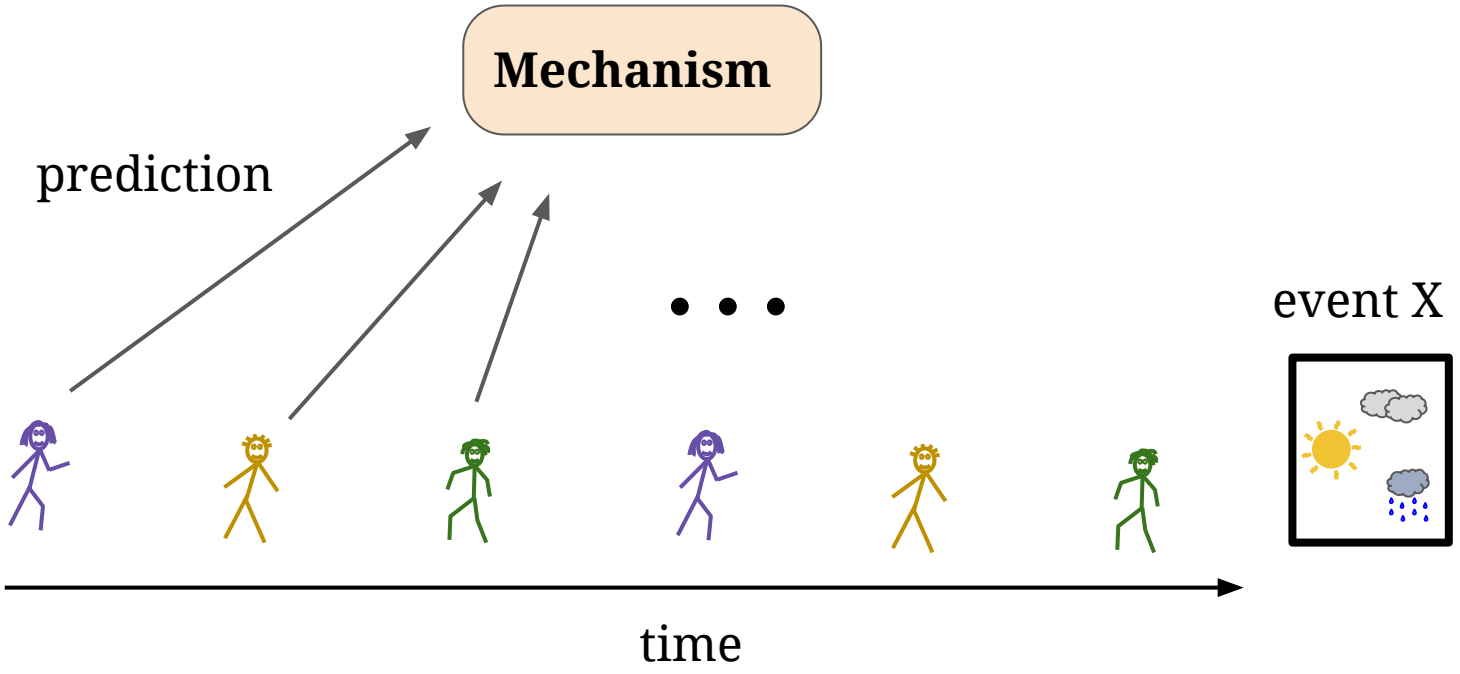
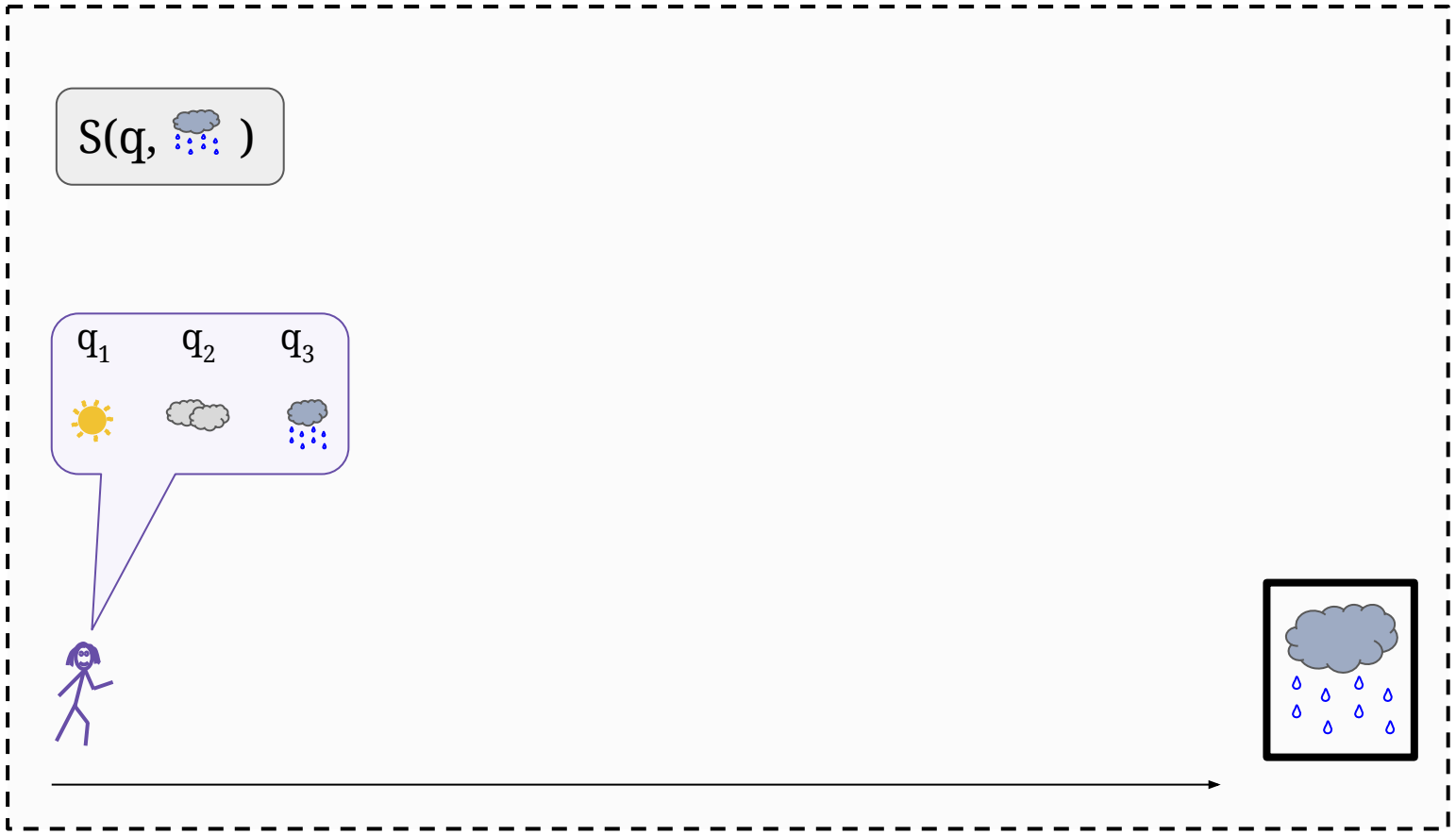


Market scoring rule [Hanson 2003]*

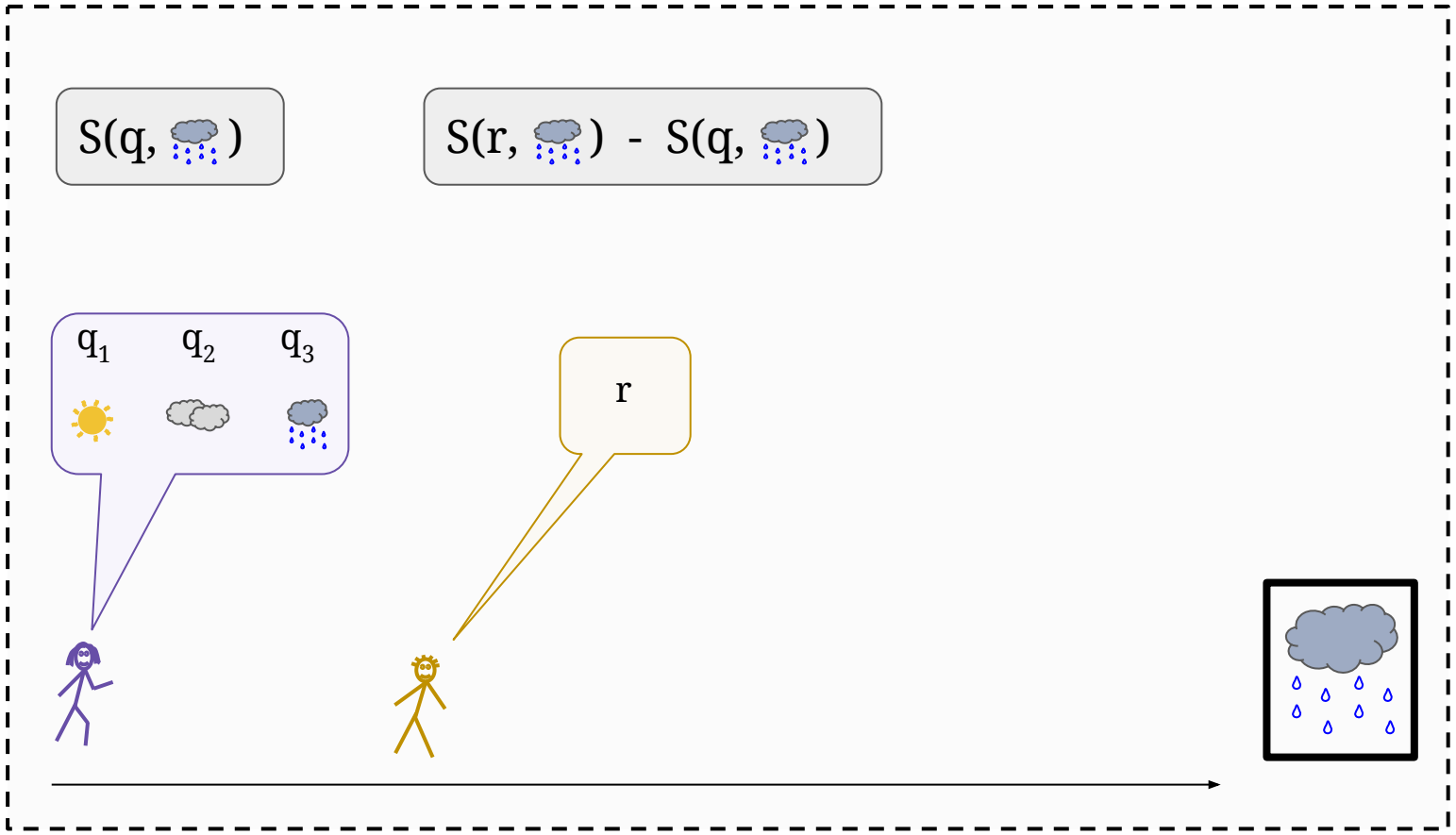
Only one participant: proper scoring rule! Truthful.



*can also be viewed as buying/selling shares [Abernethy+Chen+Wortmann-Vaughan 2013]

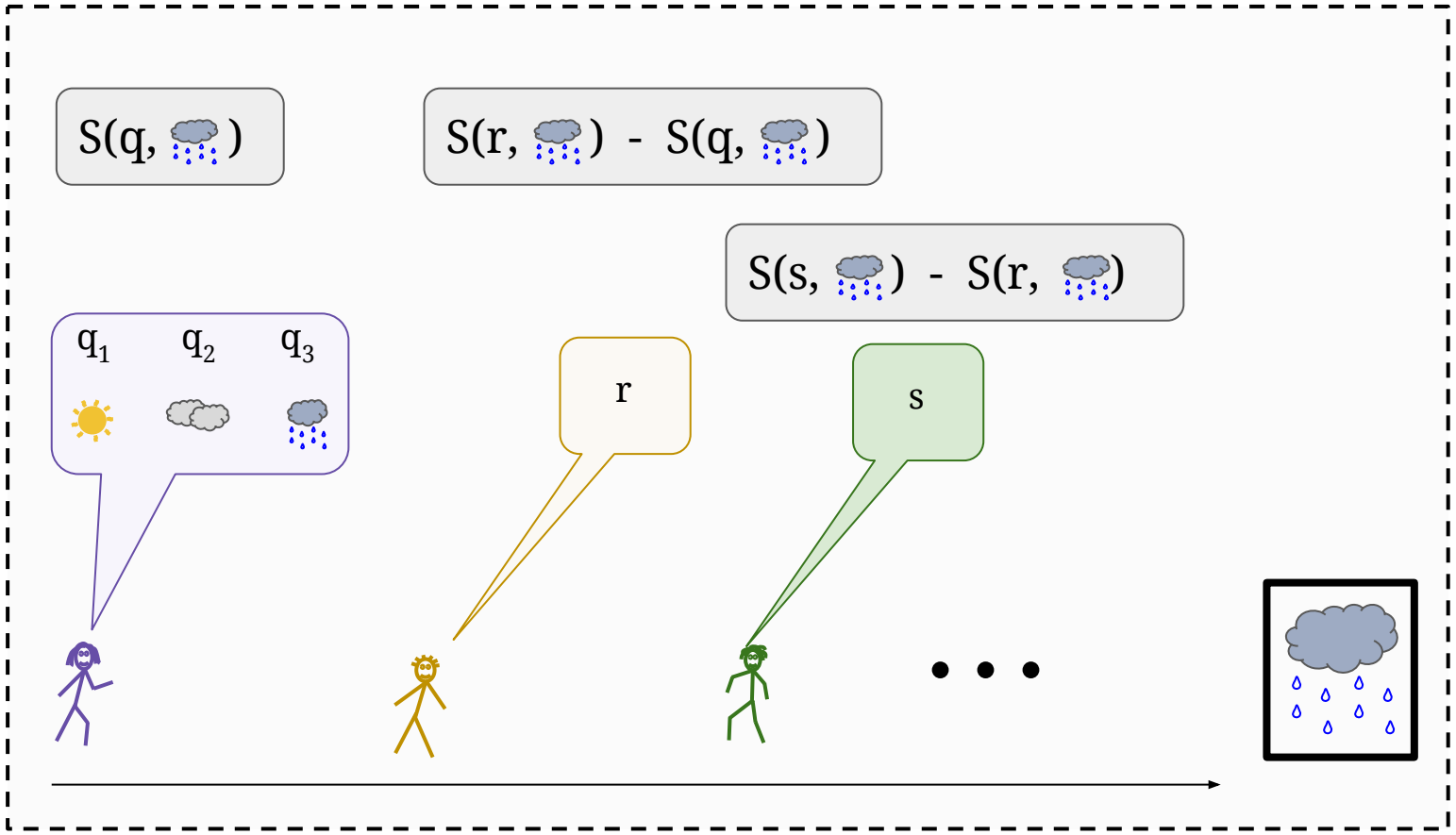
Market scoring rule [Hanson 2003]*

Two participants: “chained” scoring rule! Truthful.



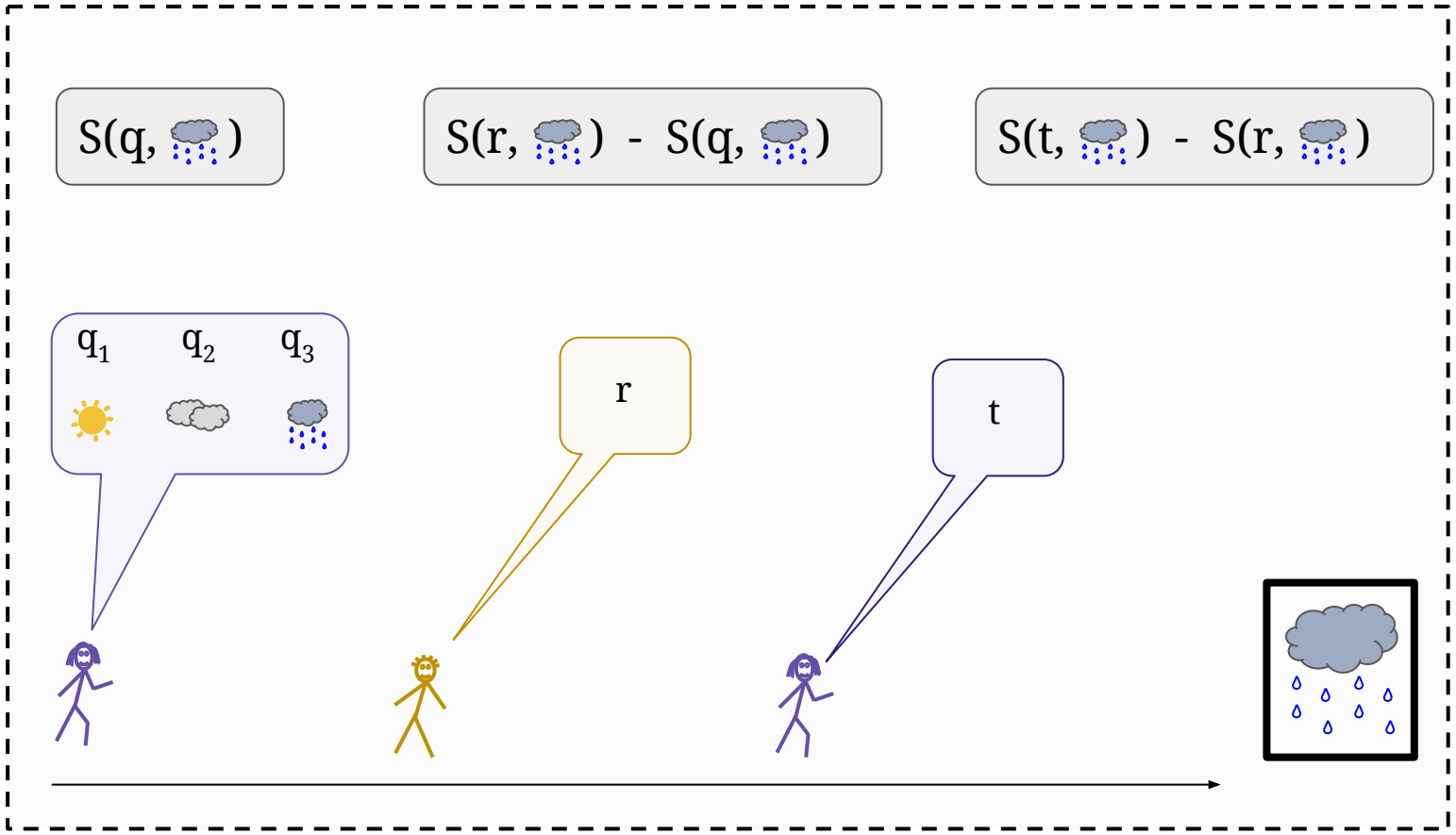
Market scoring rule [Hanson 2003]*

Many participants, each arriving only once: truthful.



Market scoring rule [Hanson 2003]

2 participants, 3 stages: **equilibria not understood!**



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Value of information [Howard 1966]

1. Known prior p on X

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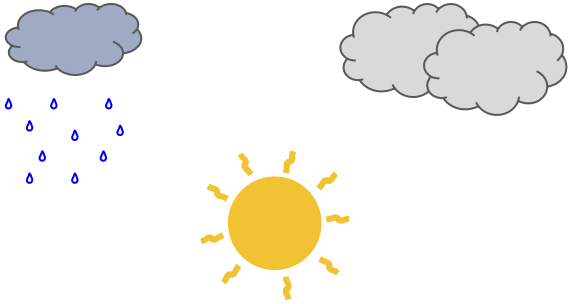
2. Select decision d

2. Nature draws x ~ p



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3. Get utility u(d, x).

V(∅) = “expected utility when deciding optimally with **no signals**”

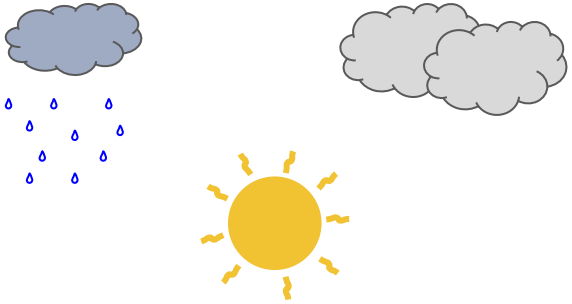
Value of information [Howard 1966]

1. Known prior p on X

1.5. **Observe Y**, Bayesian update to p_y

2. Select decision d

2. Nature draws x ~ p_y



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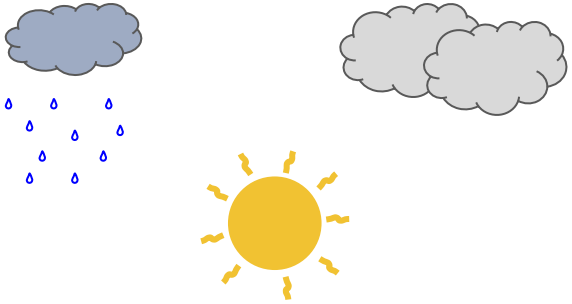

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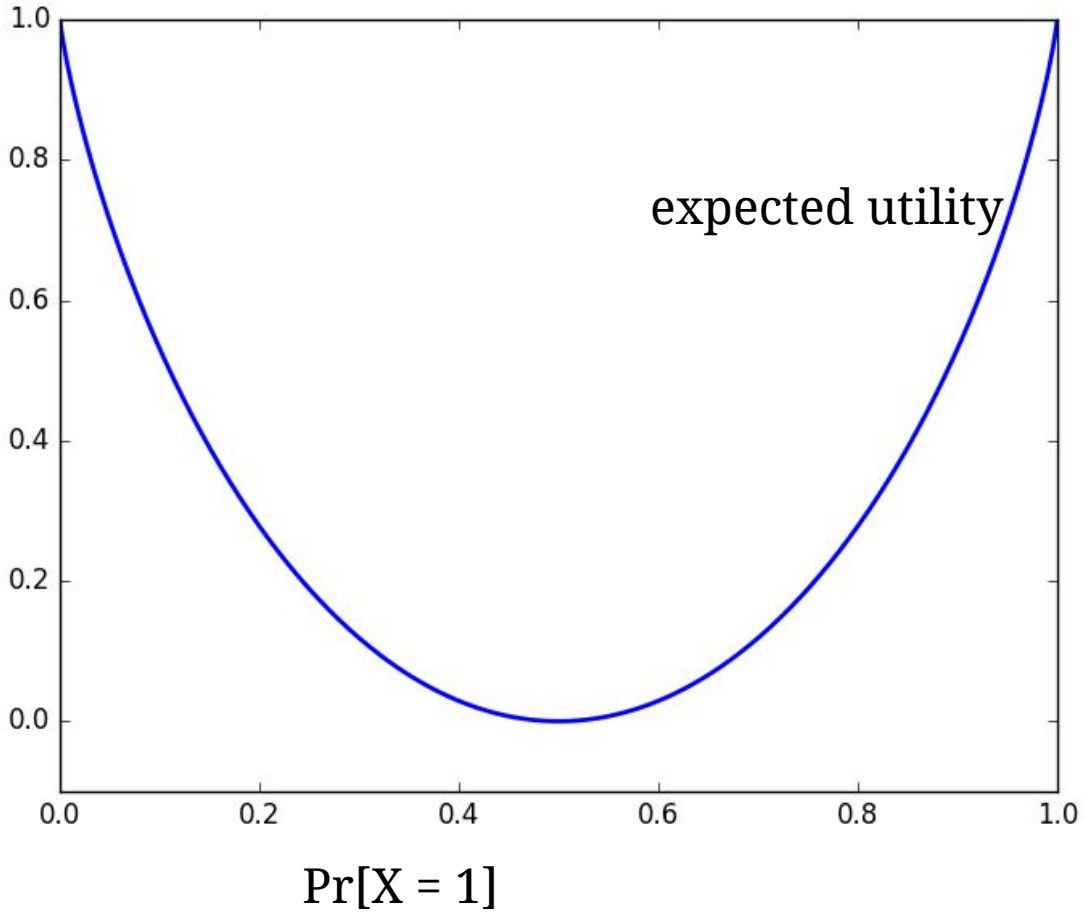
V(**Y**) = "expected utility when deciding optimally after **observing Y**"

V(Y) - V(∅) = "marginal value of Y"

Decision problems ←→ convex functions!

Example: proper scoring rule S(q, x) = log q(x).



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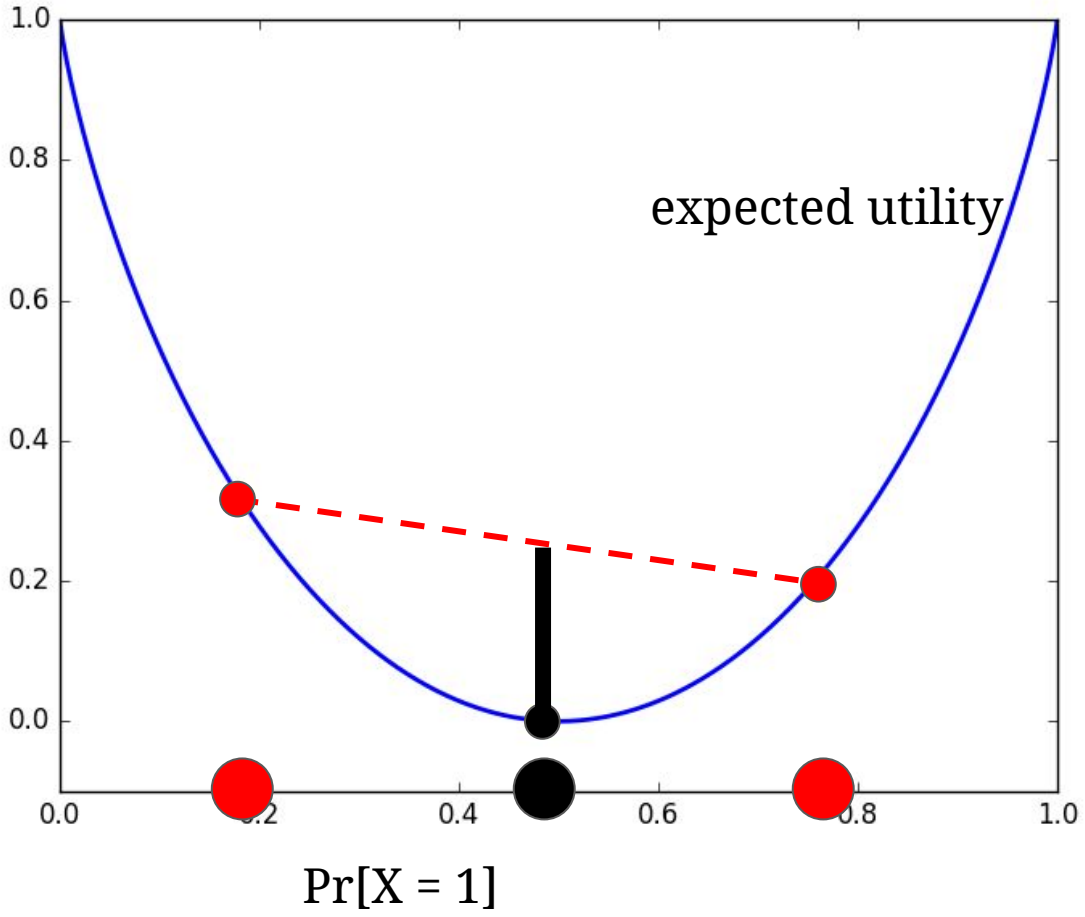


prior p on X

posteriors p_y

V(Y) - V(∅)

Our definitions

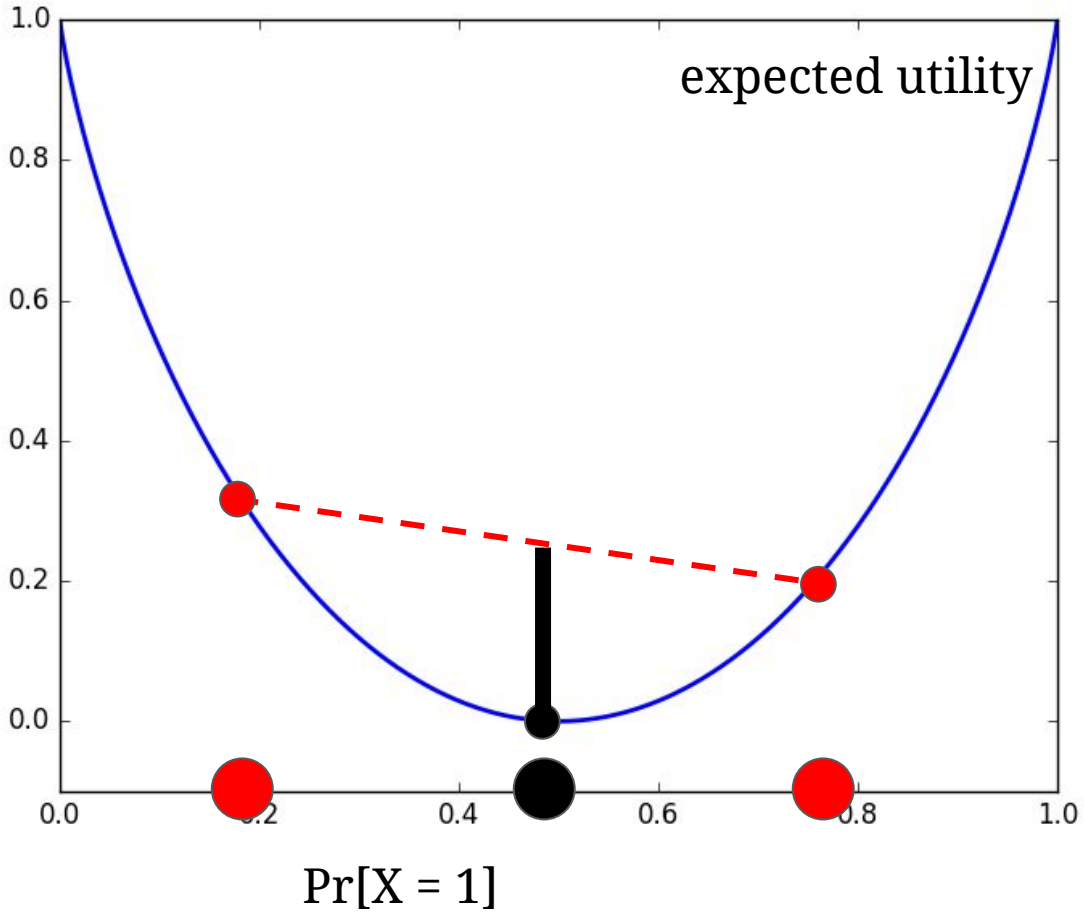
$Y_1 \ldots Y_n$ are **substitutes** for u if V is **submodular**:
For $A \subseteq B \subseteq \{Y_1 \ldots Y_n\}$,

$$V(A \cup \{Y_i\}) - V(A) \geq V(B \cup \{Y_i\}) - V(B).$$

- complements = supermodular
- depends on **both decision prob AND info structure**

Visualizing an example of substitutes

Example: S(q, x) = log q(x). Y_1, Y_2 i.i.d. conditioned on x.



prior p on X

posteriors p on one signal

V(Y) - V(∅)

Visualizing an example of substitutes

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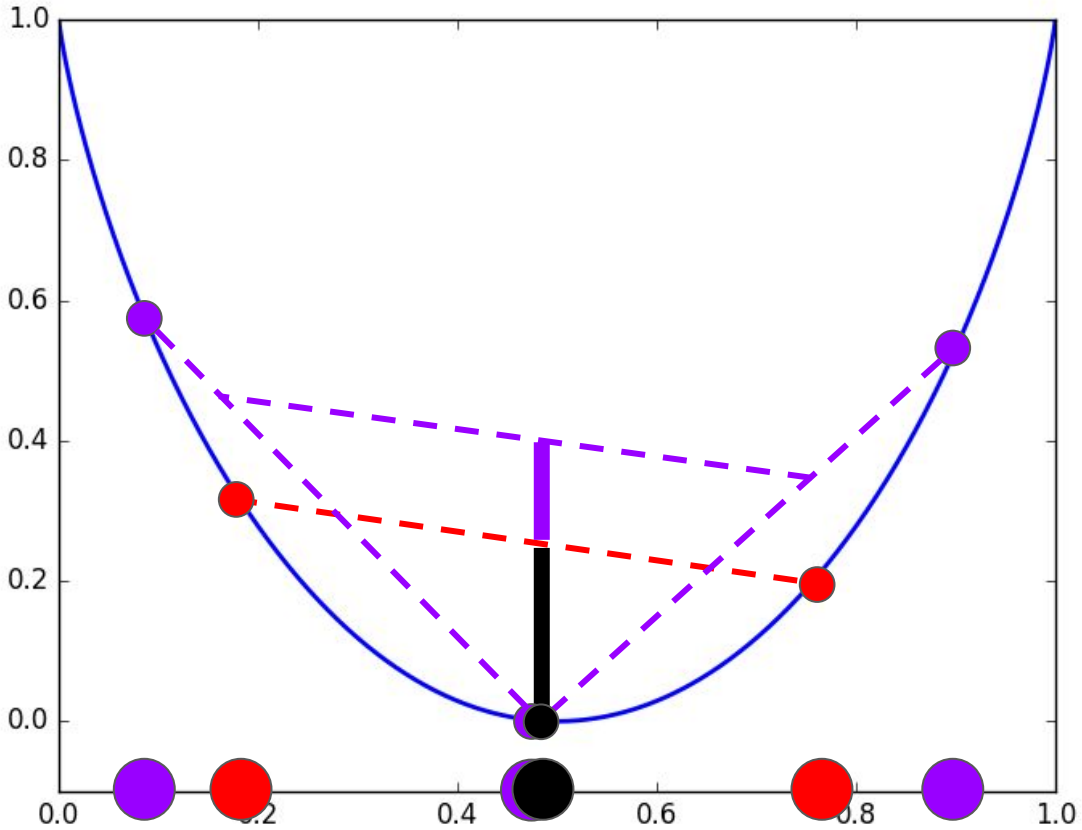


prior p on X

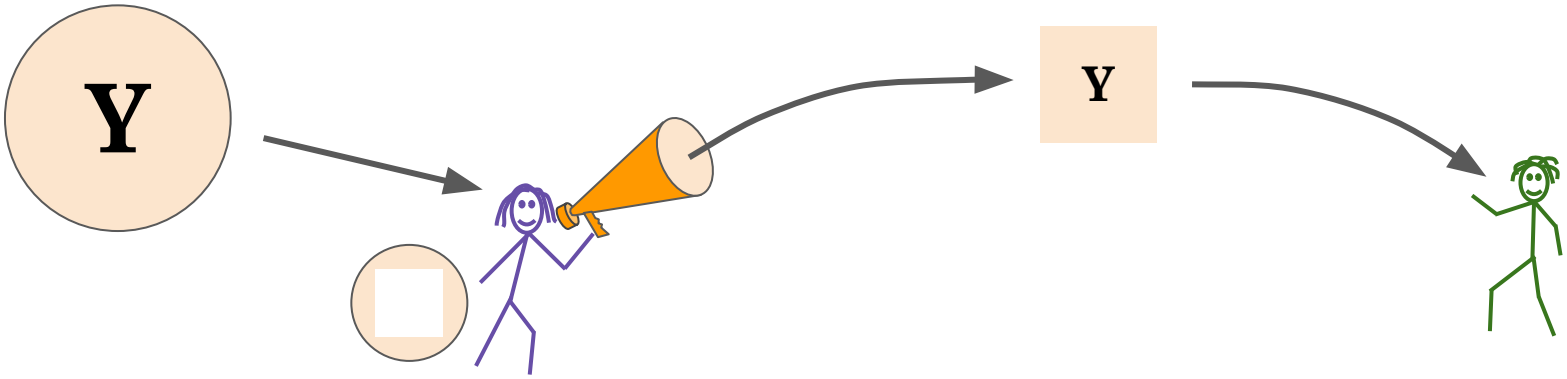
posteriors on one signal

$V(Y_1) - V(\varnothing)$

posteriors on two signals

$V(Y_1,Y_2) - V(Y_2)$

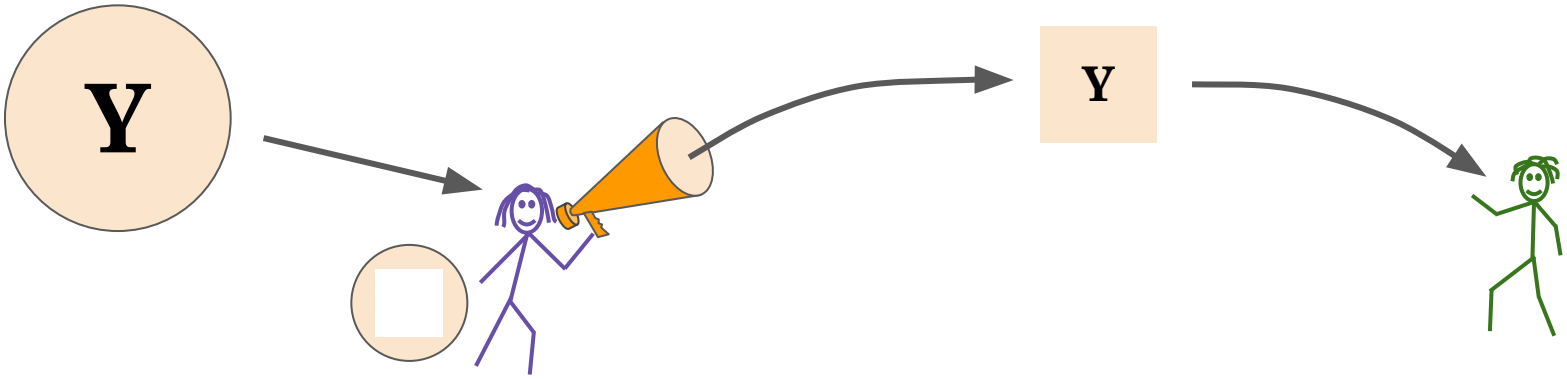
Roadblock: Information is divisible!



“Half the truth is often a great lie.”

- Benjamin Franklin

Roadblock: Information is divisible!



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Example: Alice observes entire stock market,
but strategically reports one stock's performance.

Solution in a nutshell: require "diminishing marginal value" for "pieces" of information.

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Known results on prediction markets

Known: for **log scoring rule**, if $Y_1 \ldots Y_n$ are…

- conditionally independent on X ⇒ “rush”.
  [Chen+Dimitrov+Sami+Reeves+Pennock+Hanson+Fortnow+Gonen 2010]

- independent ⇒ “delay”.
  [Gao+Zhang+Chen 2013]

Prediction markets results

Thm. If and only if signals are strong **substitutes**, the only equilibria are "**all rush**".

(efficient market hypothesis ←→ substitutes)

Thm. If and only if signals are strong **complements**, the only equilibria are "**all delay**".

(market failure ←→ complements)

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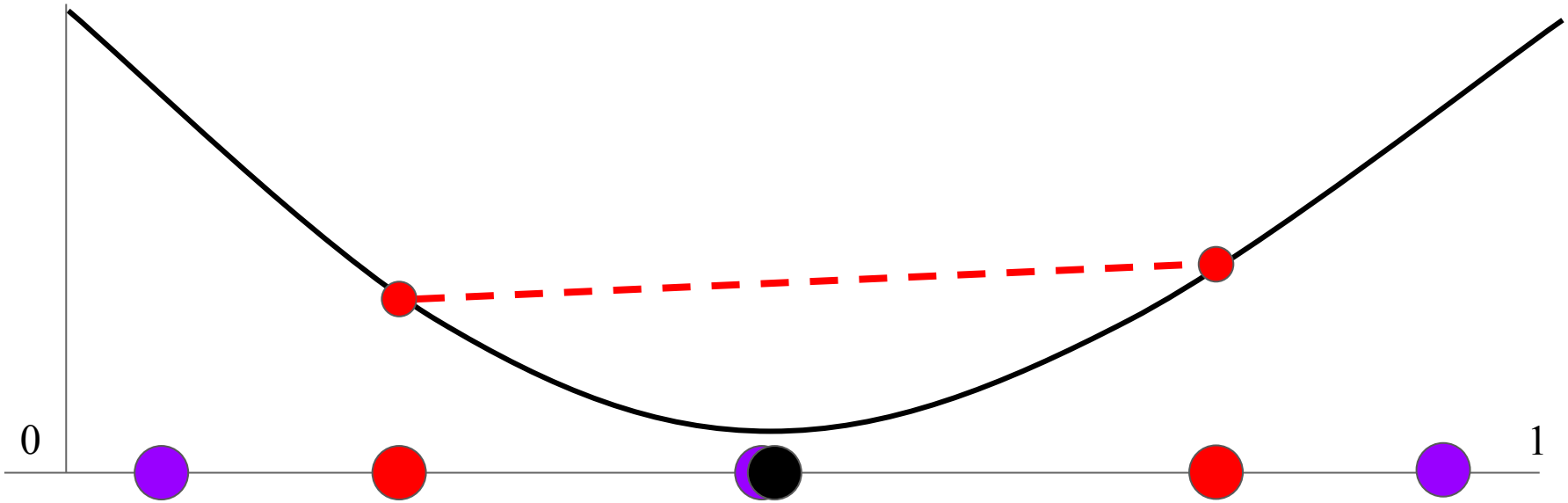
Designing for substitutes

- lots of curvature near the prior = high marginal value initially
- less curvature farther out = diminishing marginal value



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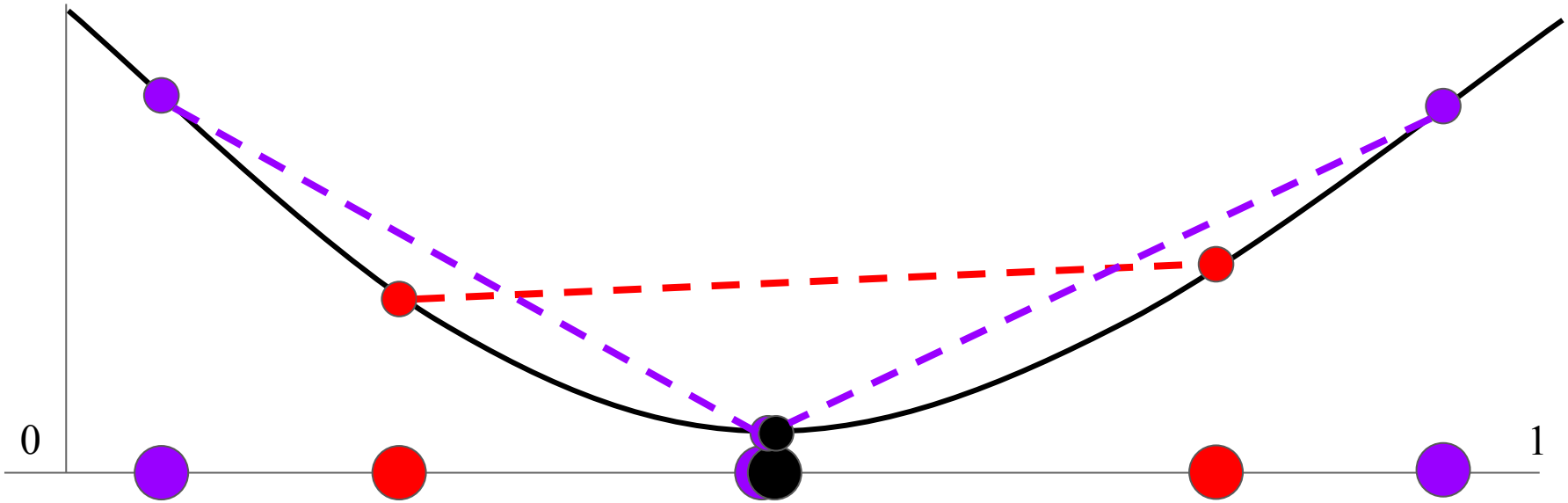
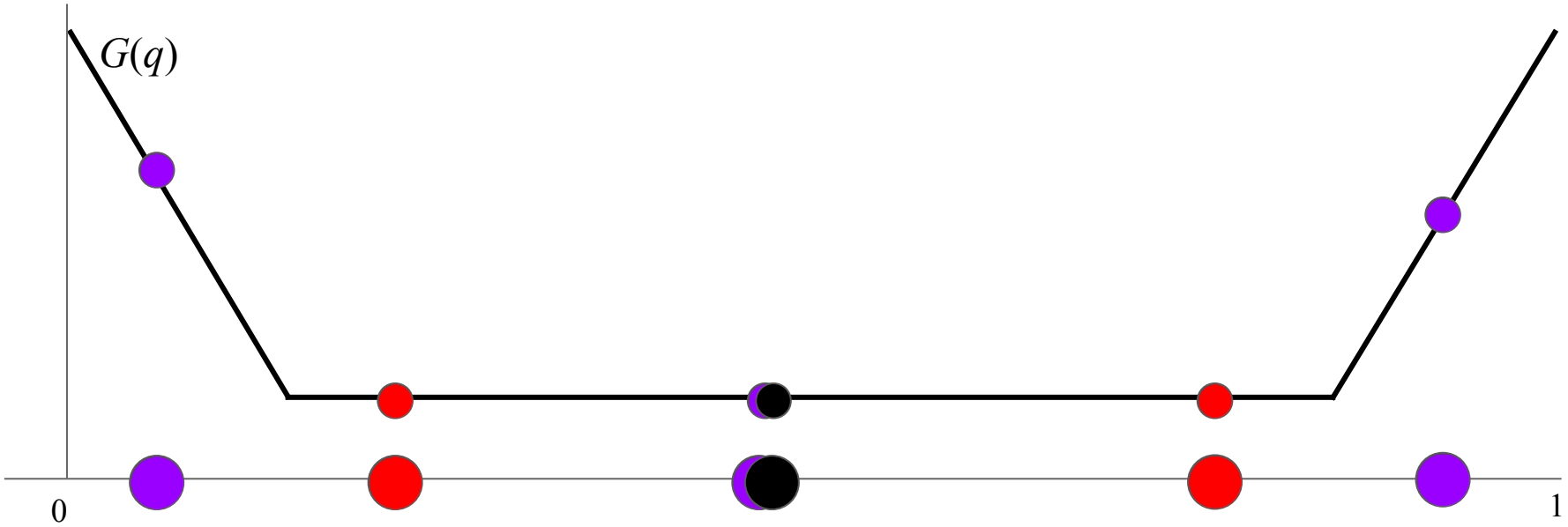


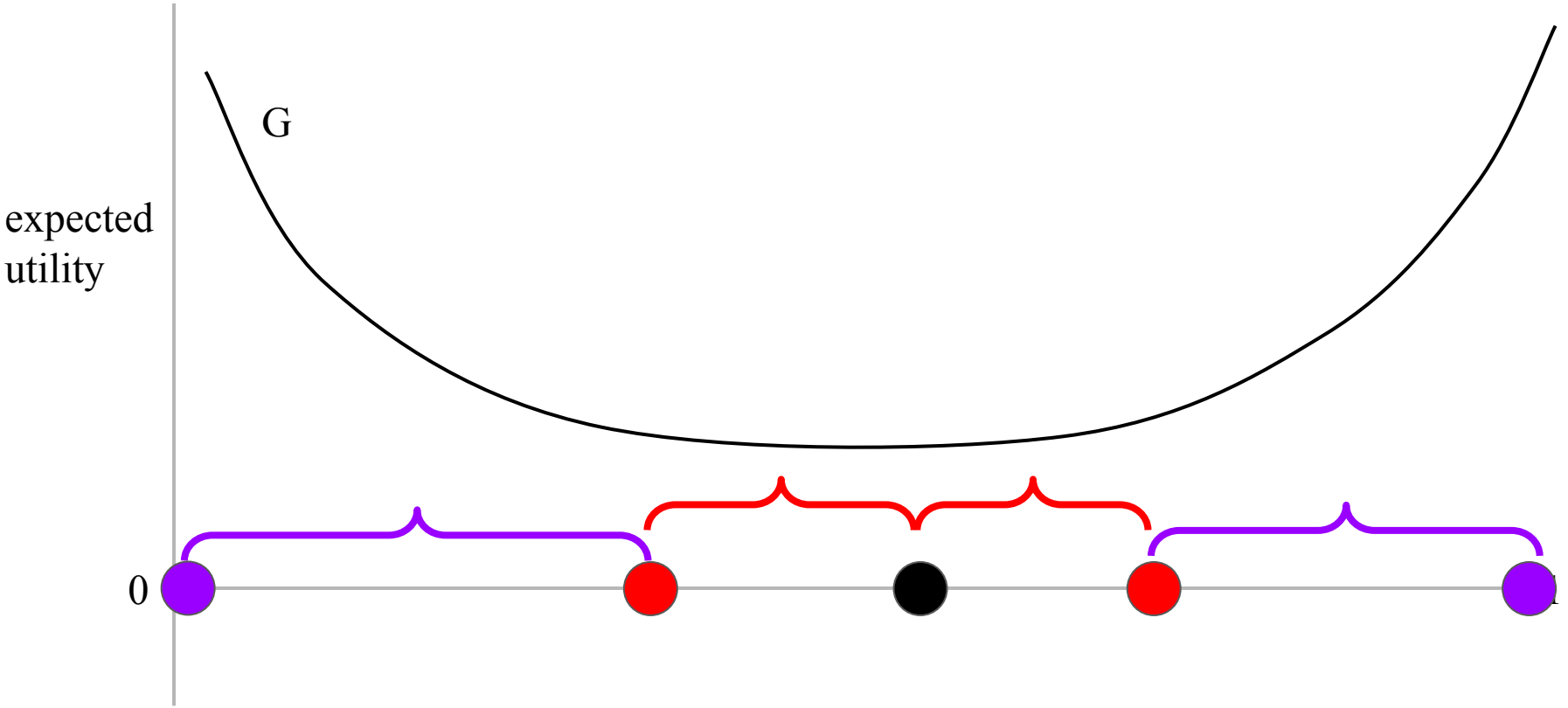
Substitutes are fragile

- For most signal structures, the **wrong scoring rule** can destroy substitutability



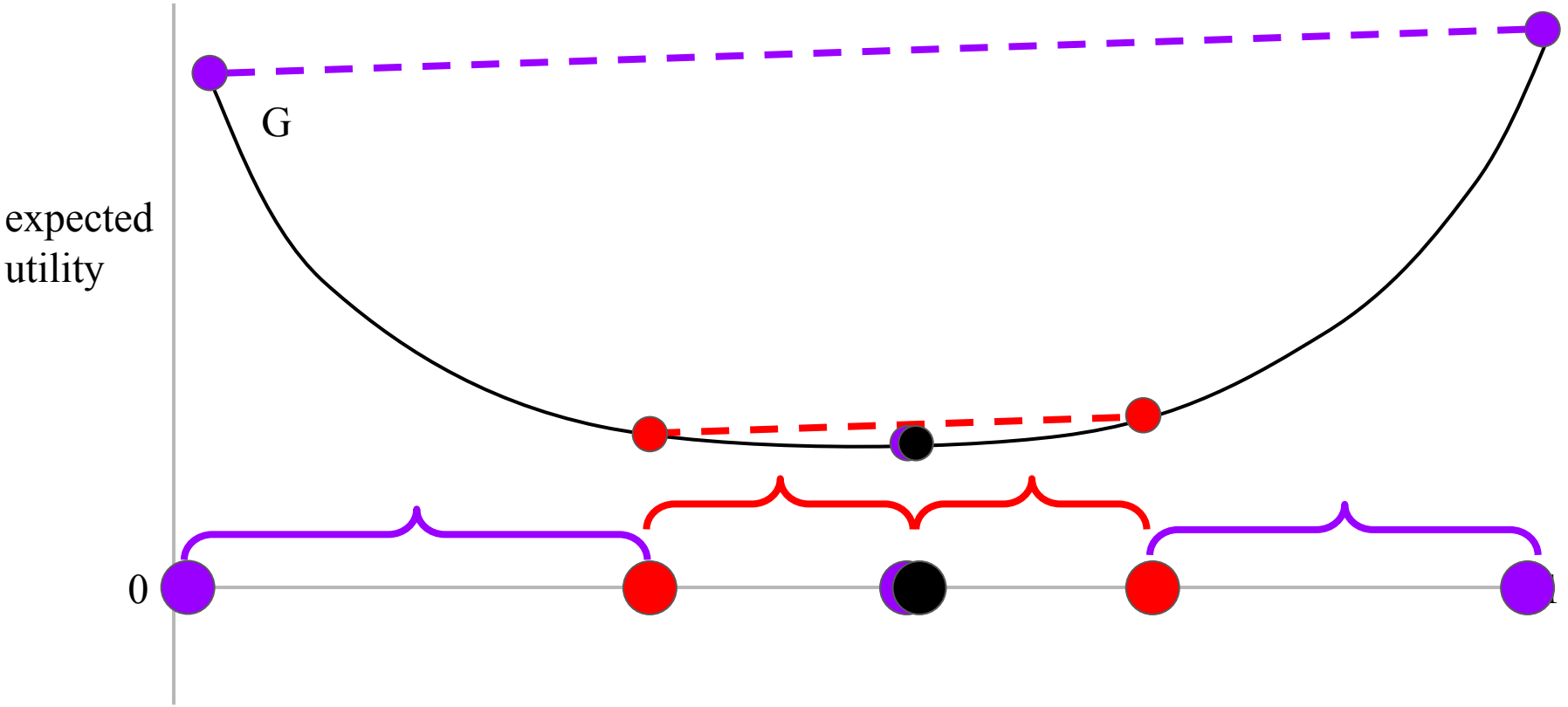
Complements are robust

- For some signal structures, it is **difficult or impossible** to design for substitutability



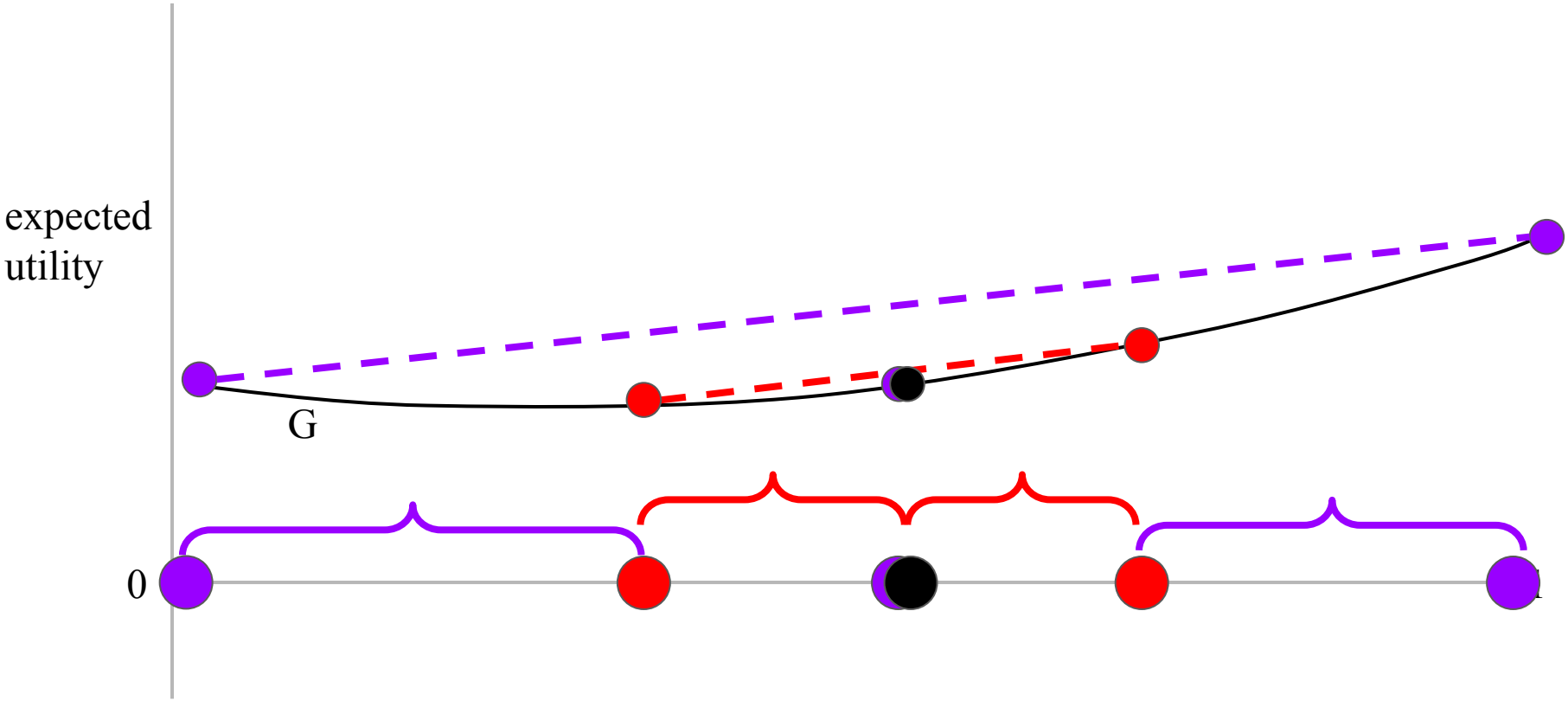
Complements are robust

- For some signal structures, it is **difficult or impossible** to design for substitutability
- Try it here!



Complements are robust

- To remove complementarity, we want to “flatten” G…
- But we can only flatten it so far!



Thanks!