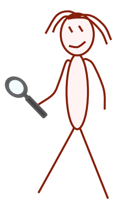
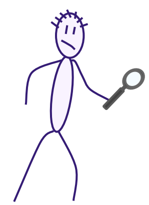
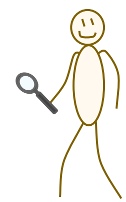
Descending Price Optimally Coordinates Search





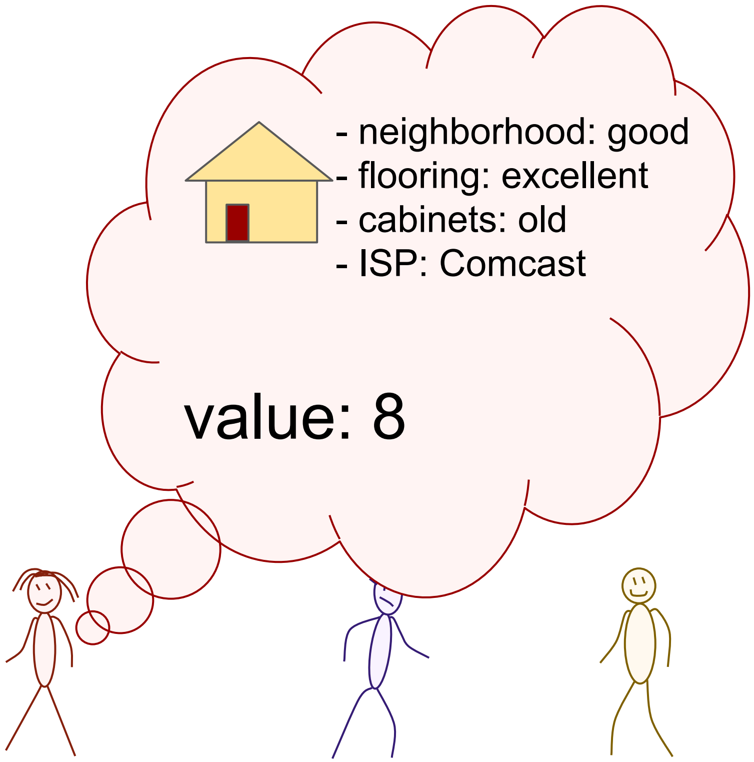


EC 2016

Robert Kleinberg…….Cornell / Microsoft
Bo Waggoner………...Harvard → UPenn
Glen Weyl…………….Microsoft / Yale

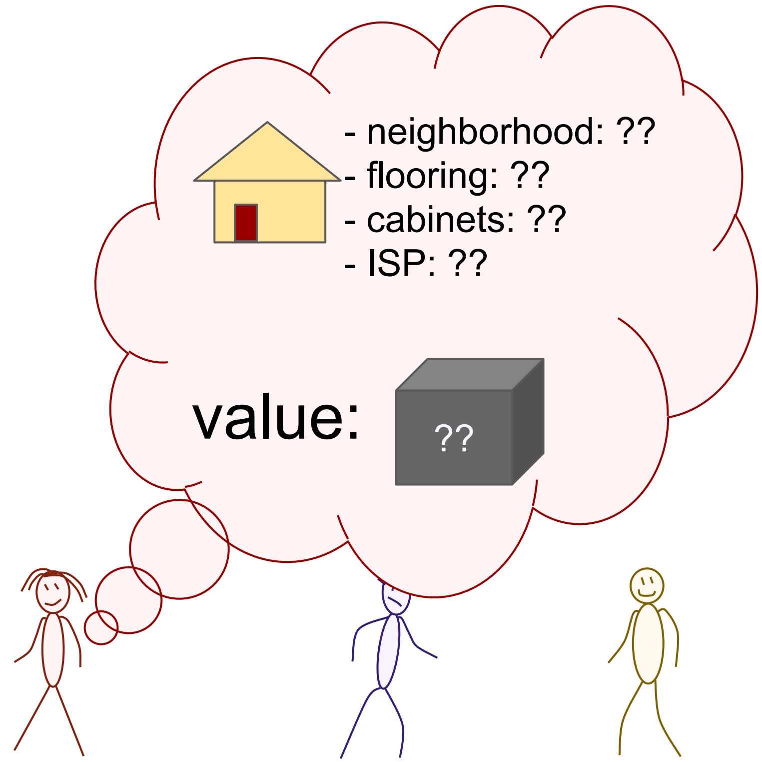
A glaring omission in mechanism design

Standard model:



fully-informed bidders

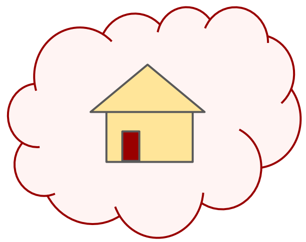
More realistic:



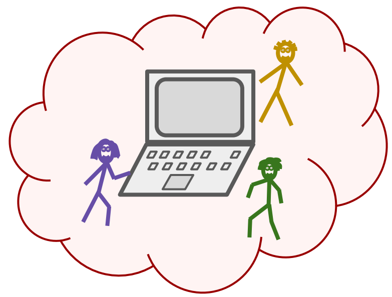
bidders must **invest effort** to learn values

Inspection costs could matter a lot:

- buying a house



- acquiring a startup



Problem: how to get good welfare?

- You'd hope traditional mechanisms would be **robust** with inspection costs

Traditional economics approaches for welfare

Since Vickrey 1961: prefer "**progressive**" procedures.

1. Begin with all potential matches.
2. Gradually discard low-value matches.
3. Eventually make high-value matches.

Examples:

- Ascending-price / second-price auctions
- Deferred acceptance

Traditional economics approaches for welfare

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Examples:

- Ascending-price / second-price auctions
- Deferred acceptance

Problems (intuitively):

- Agents must decide whether to inspect **early.**
- Bidder inspection may be **poorly coordinated.**

Our general theme

With inspection costs, mechanisms for assignment should:

1. Begin with **no** potential matches (high value threshold).

2. Allow bidders to search for highest-value matches first.

3. As soon as a match is found, lock it in.

Why (intuitively):

1. Allow bidders to search without exposure to risk.
2. Coordinate search from highest “potential value” down.

Contributions

1. Simultaneous/ascending formats are **highly suboptimal** (unbounded price of anarchy) with inspection costs.

2. On the other hand, **descending-price** correctly coordinates bidder search.

3. Combining **optimal search theory** with **auction theory** ⇒ tight correspondence to the setting without inspection.

Outline of talk

- Formal model
- The optimal search procedure
- Descending-price reduction and results
- List of extensions

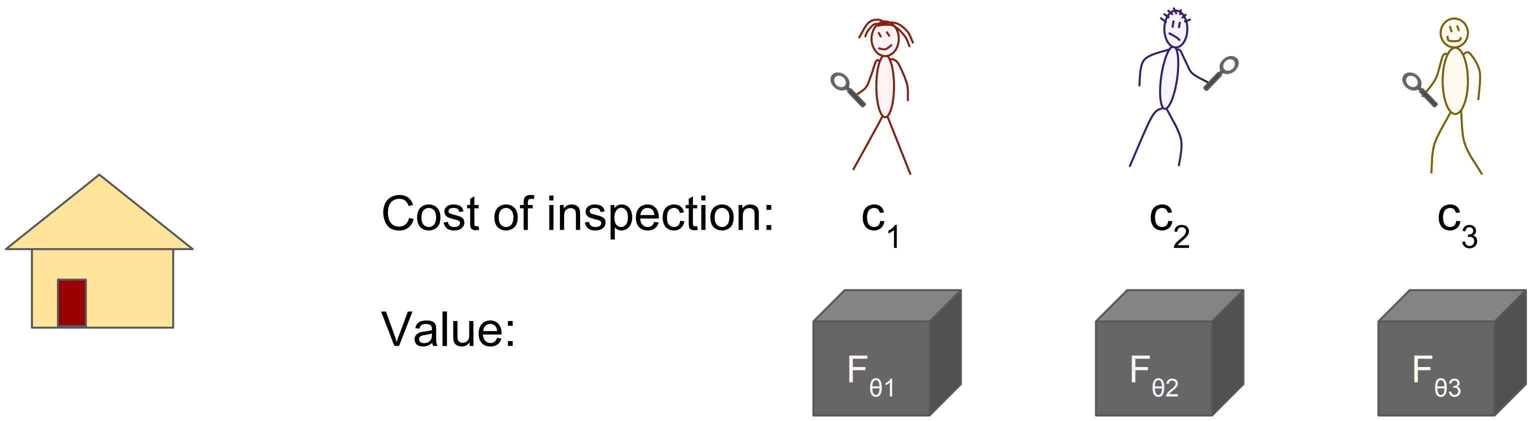
Formal model

Each j initially draws private cost c_j and type θ_j (agents may be correlated).

At any time, j may inspect, paying c_j and drawing $v_j \sim F_{\theta j}$ independently.

Inspection is:

- instantaneous,
- unobservable,
- mandatory upon obtaining the item.

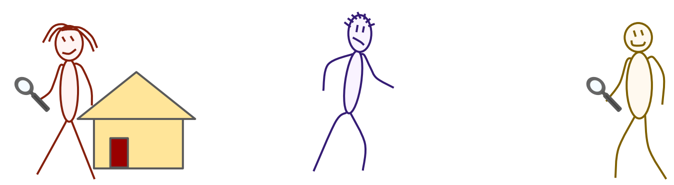


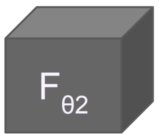
Formal model

Our goal: a mechanism with good **welfare**.

welfare = (value of winner) - (sum of all inspection costs invested)

e.g. $v_1 - c_1 - c_3$



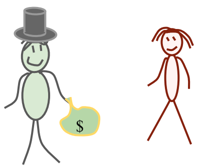
Cost of inspection:	c_1	c_2	c_3
Value:	v_1	 $F_{\theta 2}$	v_3

The Optimal Algorithm

With non-strategic bidders, solved by **Weitzman (1979)**.

Our analysis based on Gittins index theory
(**Gittins 1970s**; **Weber 1992**).

A thought experiment for bidder j



Imagine: when j inspects, an **investor** pays the inspection cost.
But: j can only keep a “capped” amount of the value; repays excess.

Suppose j **claims above the cap:** always acquires if she sees v_j > cap.
Then investor gets E[(v_j - “cap”)$^+$] .

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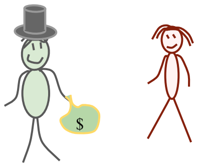

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Fair cap: $E[(v_j - \text{“cap”})^+] = c_j$.
Let $\boldsymbol{\kappa_j} := \min(v_j, \text{fair cap})$ be j’s **capped value**.

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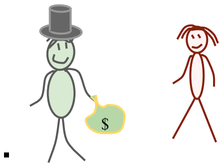

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In this imaginary scenario:

- welfare(j) = $1_j^{acq} \kappa_j$ (because j pays 0 cost and gets k_j for item)

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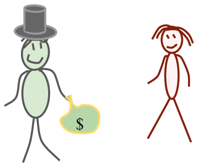

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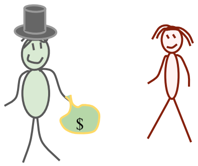

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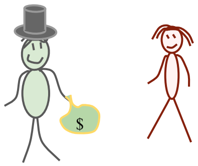

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In this imaginary scenario:

- welfare(j) = $1_j^{acq}\, \kappa_j$ (because j pays 0 cost and gets k_j for item)
- welfare(j) = reality (if j claims above the cap)
- welfare(j) $\geq$ reality (otherwise).

Key Lemma: welfare(j) $\leq E[\, 1_j^{acq}\, \kappa_j \,]$ with equality if j claims above the cap.

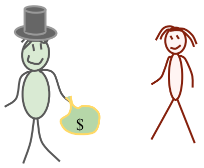
Deriving OPT

Key Lemma: welfare(j) ≤ E[1_j^{acq} κ_j]   with equality if j claims above the cap.

Corollary 1: welfare(OPT) <= E[$\max_j \kappa_j$].

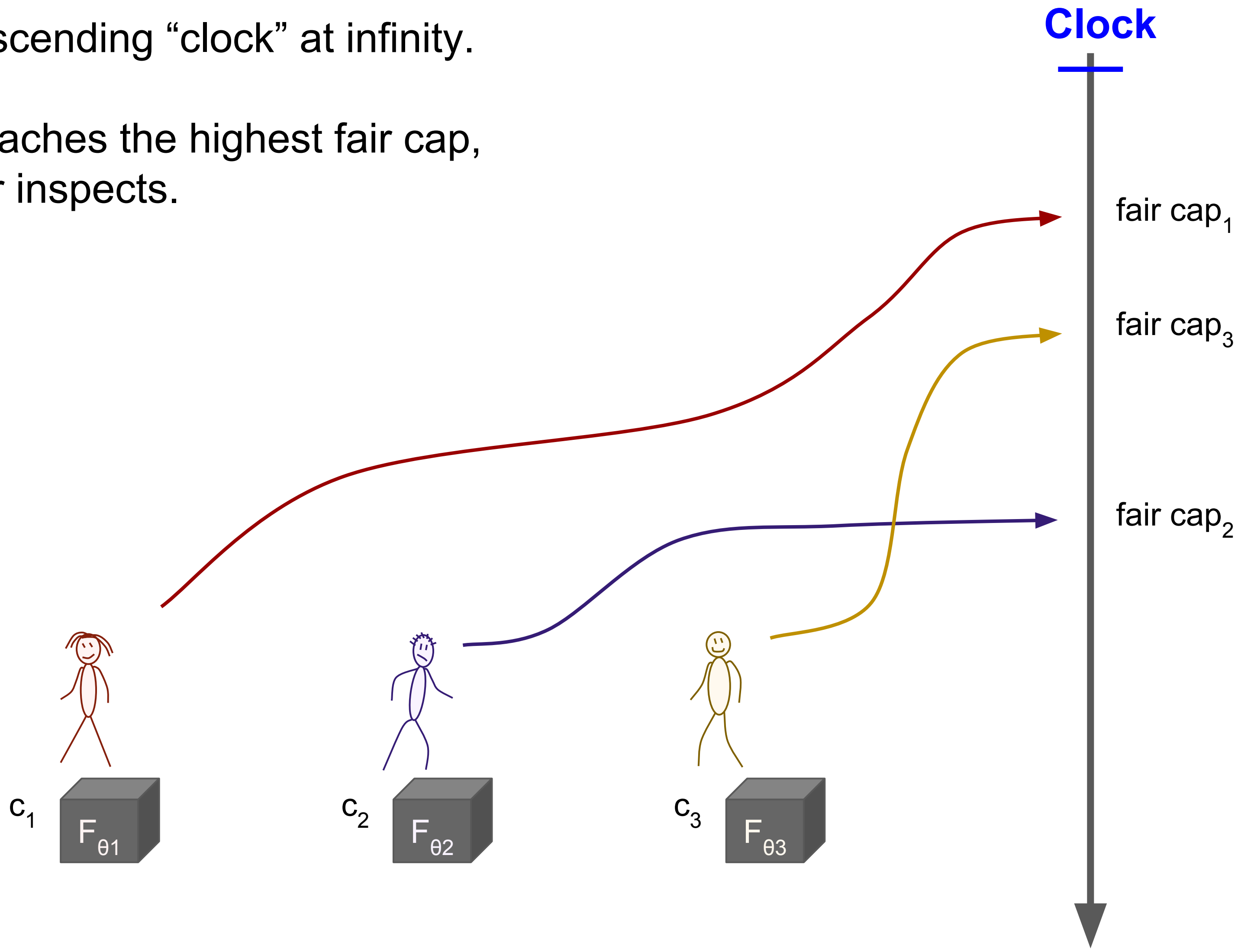
Corollary 2: Always allocating to $\text{argmax}_j \kappa_j$ is optimal...

...if all bidders claim above the cap.



The optimal algorithm

1. Start a descending "clock" at infinity.
2. When it reaches the highest fair cap, that bidder inspects.



The optimal algorithm

1. Start a descending "clock" at infinity.

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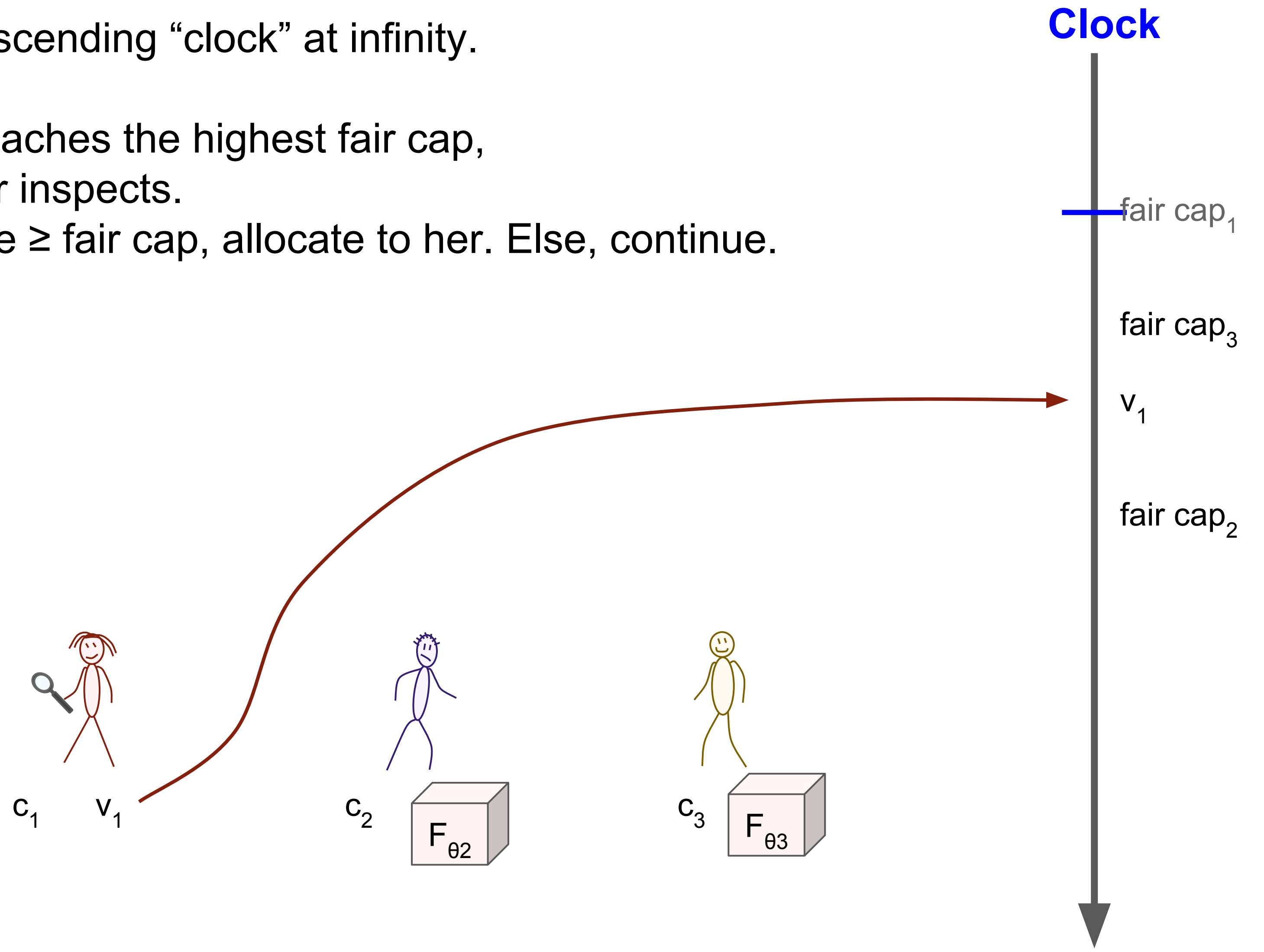

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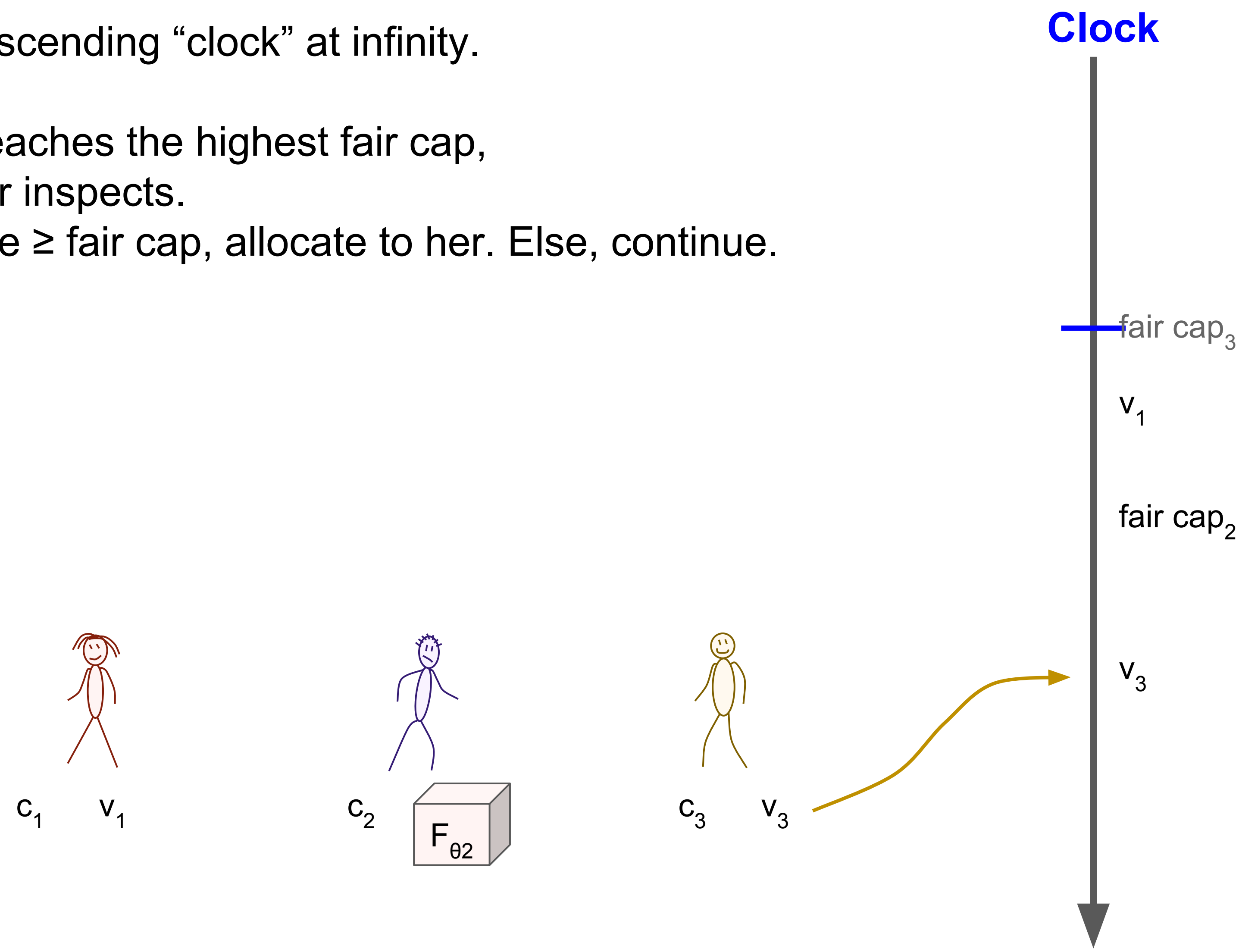

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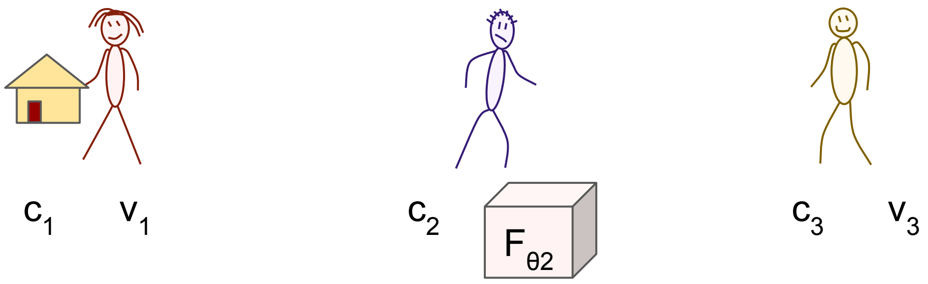

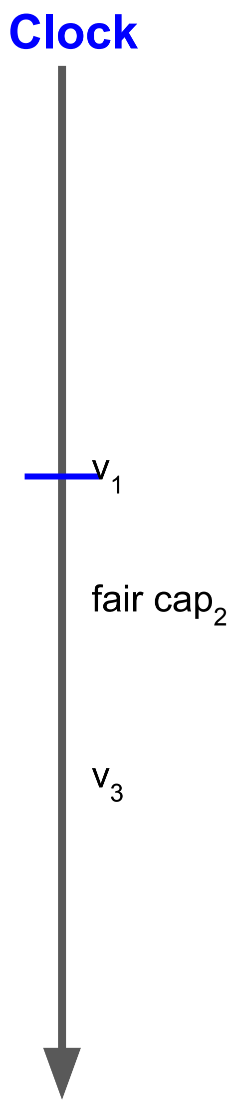

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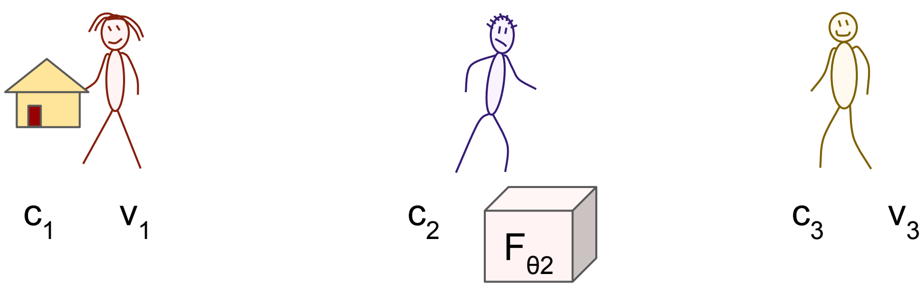
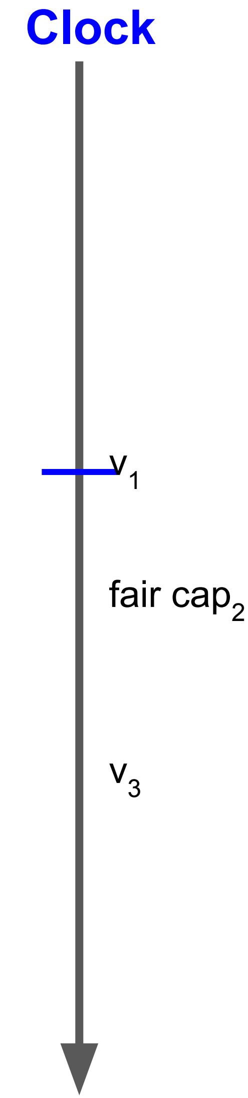
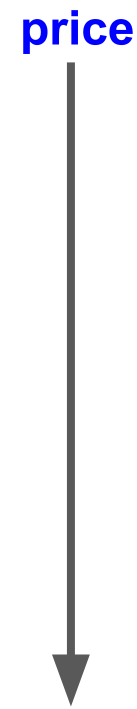
Check: bidders always claim above the cap,
allocated to highest κ_j.





From Algorithm to Mechanism



Descending-price:

- Global descending price starting from infinity.
- At any time, any bidder may claim the item, ending the auction and paying the current price.

Main results: reduction to classic first-price

Theorem: The best-response "claim time" and welfare of:

- a bidder with capped value κ, and
- a bidder with zero inspection cost and value equal to κ

are identical. Furthermore, bidders claim above the cap.

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Corollaries:

- Equilibria are in one-to-one correspondence with first-price
- e/(e-1) price of anarchy
- optimal welfare when bidders are "symmetric"
- … any other property of first-price auctions.

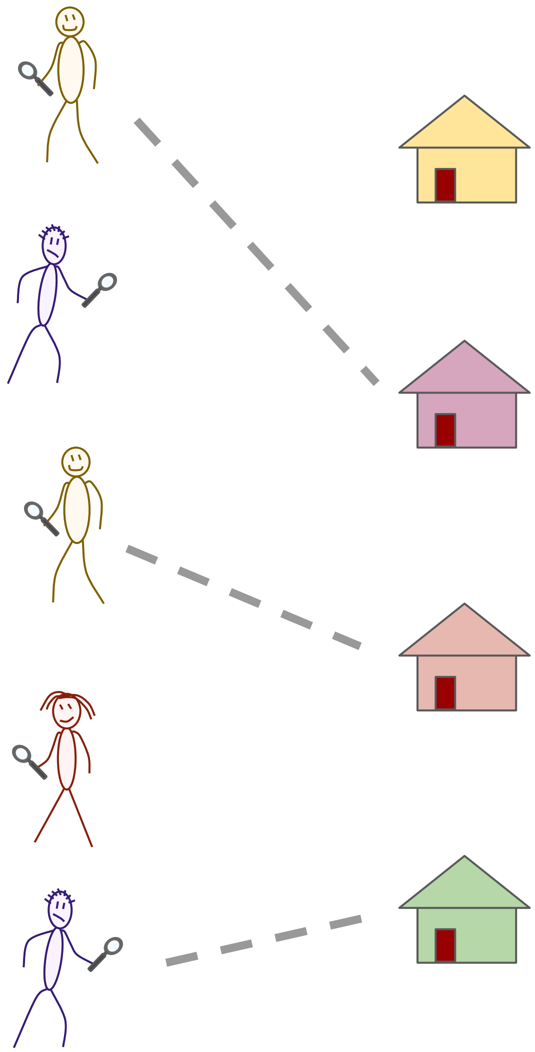
In other words: the Dutch auction is **invariant to inspection costs.**

Why? Bidders claim above the cap;
can act as though funded by an investor.
→ **Minimizes exposure to risk.**

Extension: multi-item assignment

Multi-item, unit demand setting:

- Global descending clock; claim any item any time.
- Welfare >= 0.43 * opt.
  (note: Gittins fails! 0.5+ε in polytime unknown)
- We don’t know if bidders claim above the cap, but they have a smoothness deviation that does.



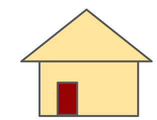
Recall: Vickrey fails even with a single item!
Key principles the same:

- Coordinate search from high to low (across items and bidders).
- Minimize exposure to risk.

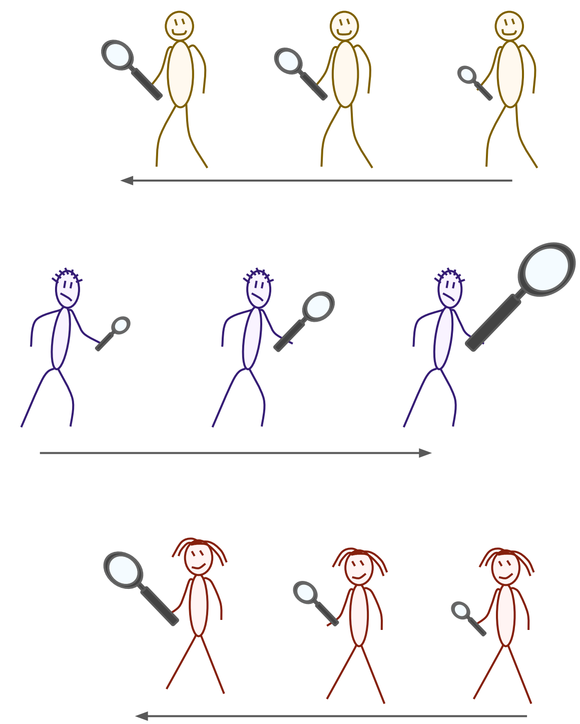
Other extensions

- Multiple **stages** of inspection (no loss in welfare!).

- Sequential posted-price also achieves a constant factor under independence (using prophet inequality).



- Common values.
- Revenue guarantee.
- Approximate best-responses.



Key theme: if bidders claim above the cap, analysis essentially reduces to standard setting.

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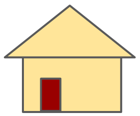

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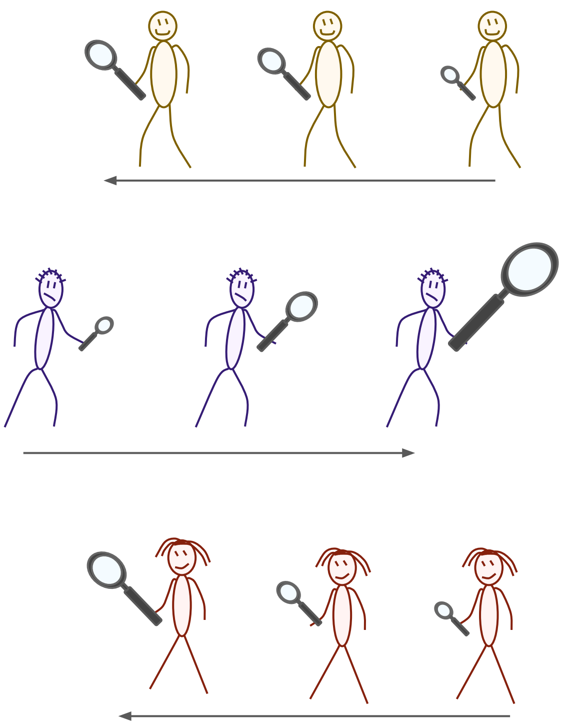

Thanks!

Excess slides

Some notes on the fair cap

1. The fair cap measures the "potential value" of each bidder.

2. Explore "high-risk, high-reward" options first.